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Abraham A. Ungar's Autobiography

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Abstract

This autobiography presents the scientific living of Abraham Ungar and his role in Gyrogroups and Gyrovector spaces.

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Figure 1: Abraham Ungar at 2016.

Abraham Ungar is professor in the Department of Mathematics at North Dakota State University. After gaining his B.Sc. (1965) and M.Sc. (1967) from

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the Hebrew University in Pure Mathematics and Ph.D. (1973) from Tel-Aviv University in Applied Mathematics, he held a postdoctoral position at the University of Toronto (1974). Ungar moved from Toronto to Pretoria where he held the position of a senior research officer at the National Research Institute for Mathematical Sciences of the Council for Scientific and Industrial Research (CSIR) in 1975-1977. From Pretoria Ungar moved to Grahamstown, South Africa, where he held the position of a lecturer and a senior lecturer at Rhodes University (1978-1983). From Grahamstown Ungar moved to Vancouver, where he held the position of a visiting associate professor at Simon Fraser University (1983-1984). Finally, in 1984 Ungar has accepted the position of an associate professor at North Dakota State University in Fargo, North Dakota, where he presently holds the position of a professor.

Ungar's favored research areas are related to hyperbolic geometry and its applications in relativity physics. He currently serves on the editorial boards of *Mathematics Interdisciplinary Research*, of *Journal of Geometry and Symmetry in Physics*, and of *Communications in Applied Geometry.*

When Ungar was a young, undergraduate student he was fascinated by the bijective correspondence between the field of complex numbers and the Lorentz transformation group of special relativity theory in one time and one space dimensions. He was aware of the result that the field of complex numbers does not admit extension to a field of higher than two dimensions while, in contrast, the Lorentz group admits extensions to one time and several space dimensions. Ungar, therefore, felt that the transition of the Lorentz group from one time and one space dimensions, where it is closely related to the field of complex numbers, to one time and two space dimensions is a mystery to be conquered. Hence, later young student Ungar was not surprised to discover in the literature that a new phenomenon comes into play in the above mentioned transition of the Lorentz group. The new phenomenon, which deeply attracted Ungar's attention, turned out to be the peculiar space rotation known in special relativity theory as *Thomas precession.*

Naturally, many explorers were fascinated by the relativistic Thomas precession. However, the elegant structure that Thomas precession encodes could not be decoded for a long time, being locked by complexity. Indeed, the hopeless status of Thomas precession that existed before Ungar's 1988 discovery is well described by Herbert Goldstein in his book *Classical Mechanics* (Addison-Wesley, 1980, pp. 285-286):

> The decomposition process can be carried through on the product of two pure Lorentz transformations to obtain explicitly the [Thomas] rotation of the coordinate axes resulting from the two successive boosts. In general, the algebra involved [in calculating the Thomas rotation] is *quite forbidding,* more than enough, usually, to *discourage any actual demonstration* of the rotation matrix. There is, however, one specific situation where allowable approximations reduce the calculational com-

> plexity, while the result obtained has important applications in many areas of modern physics. What is involved is a phenomenon known as the *Thomas precession*.
>
> Herbert Goldstein, Classical Mechanics

In 1988 Ungar expressed the Lorentz group parametrically in terms of relativistically admissible velocities and orientations in an article titled *Thomas Rotation and the Parametrization of the Lorentz Transformation Group*. The group structure of the resulting parametric realization of the Lorentz group, along with Einstein velocity addition law, enabled Ungar to discover the rich structure that Thomas precession possesses in terms of Einstein addition.

Revealing its underlying structure, Ungar was able to extend Thomas precession by abstraction, calling the abstract Thomas precessions *gyrations*. It turned out that gyrations are automorphisms that regulate Einstein addition in the sense that the seemingly structureless, noncommutative, nonassociative Einstein addition is, in fact, a gyrocommutative, gyroassociative binary operation in a gyrocommutative gyrogroup and in a gyrovector space. The ugly duckling of relativity physics, Thomas precession, thus became the beautiful swan called, in *gyrolanguage*, gyration.

The resulting emergence of the gyrogroup and the gyrovector space structures, along with their application in the hyperbolic geometry of Lobachevsky and Bolyai and in the special relativity theory of Einstein, is unfolded by Ungar in the following expository article titled:

The Intrinsic Beauty, Harmony and
Interdisciplinarity
in Einstein Velocity Addition Law:
Gyrogroups and Gyrovector Spaces.

Yet, the rich structure of Einstein addition and the interdisciplinarity of gyrogroups and gyrovector spaces that Ungar has exposed is still far from being exhausted, as the articles of the present special issue of the Journal demonstrate.

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