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Motion of Particles under Pseudo-Deformation

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Abstract

In this short article, we observe that the path of particle of mass m moving along $\mathbf{r} = \mathbf{r}(t)$ under pseudo-force $\mathbf{A}(t)$, t denotes the time, is given by $\mathbf{r}_d = \int(\frac{d\mathbf{r}}{dt}\mathbf{A}(t))dt + \mathbf{c}$. We also observe that the effective force $\mathbf{F}_e$ on that particle due to pseudo-force $\mathbf{A}(t)$, is given by $\mathbf{F}_e = \mathbf{F}\mathbf{A}(t) + \mathbf{L}d\mathbf{A}(t)/dt$, where $\mathbf{F} = m\ d^2\mathbf{r}/dt^2$ and $\mathbf{L} = m\ d\mathbf{r}/dt$. We have discussed stream lines under pseudo-force.

Keywords: Right loops, right transversals, gyrotransversals.

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1. Introduction

In [3], we have observed that $\mathbb{R}^n$ is a unique gyrotransversal [1] to the subgroup $O(n)$ in the group $Iso\ \mathbb{R}^n$, the group of motion (Cor 6.8 [3]). If S is a right transversal to the subgroup H of a group G and $g : S \to H$ is a map with $g(e) = e$, e being identity of G. Then, it induces a binary operation o_g [3, 4] on S is given by

$$xo_g y \quad = \quad x\theta g(y)oy. \tag{1}$$

This suggests us that the map g affects the sum xoy of $x, y \in S$ and the effective sum is $x\theta g(y)oy$ instead of xoy. Indeed, this group-theoretic idea makes certain sense.

2. Preliminaries

Let S be a non-empty set. Then, a groupoid (S, o) is called a *right quasigroup* if for each x, y in S, the equation $X\ o\ x = y$ has a unique solution in S, where X is

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unknown in the equation. If there exists $e \in S$ such that $eox = x = xoe$ for every $x \in S$, then the right quasigroup (S, o) is called a *right loop*.

Let H be a subgroup of a group G. Then a set S obtained by selecting one and only one element from each right coset of G modulo H, including identity of G is called a *right transversal* to H in G. The group operation induces a binary operation o on S and an action θ of H on S given by $\{xoy\} = S \cap Hxy$ and $\{x\theta h\} = S \cap Hxh$ respectively, where $x, y \in S$ and $h \in H$. One may easily observe that (S, o) is a right loop with identity e, where e is the identity of group. Indeed, it determines an algebraic structure (S, H, σ, f) known as $c-groupoid$ [2]. Conversely a given c-groupoid (S, H, σ, f) determines a group $G = H \times S$ which contains H as a subgroup and S as a right transversal to H in G so that the corresponding c-groupoid is (S, H, σ, f) (Theorem 2.2, [2]). It is also observed that every right loop (S, o) can be embedded as a right transversal to a subgroup $Sym\ S \setminus \{e\}$ in to a group $Sym\ S \setminus \{e\} \times S$ with some universal property (Theorem 3.4, [2]).

Let S be a fixed right transversal to a subgroup H in a group G. Then every right transversal to H in G determines and is determined uniquely by a map $g : S \to H$ such that $g(e) = e$, the identity of G. The right transversal S_g determined by a map $g : S \to H$ is given by $S_g = \{g(x)x | x \in S\}$. The induced operations o on S and o' on S_g are given by

$$\{xoy\} = Hxy \cap S$$

and

$$\{g(x)xo'g(y)y\} = S_g \cap Hg(x)xg(y)y,$$

respectively. Further, H acts on S from right through an action θ given by $\{x\theta h\} = Hxh \cap S, \forall\, x \in S, h \in H$. Indeed, the right loop (S_g, o') is isomorphic to the right loop (S, o_g) where the binary operation o_g on S is given by $xo_gy = x\theta g(y)oy$ [3, 4]. This suggests us to say that *the map $g : S \to H$ with $g(e) = e$ affects the operation o on S and the resulting operation on S due to the effect of g is o_g, where $x\ o_gy = x\theta g(y)oy$.*

3. Pseudo-Force, Deformed Path and Effective Force

Consider the group of motion $Iso\ \mathbb{R}^n$. As we have observed in [3] that $\mathbb{R}^n$ is a right transversal (unique gyrotransversal) to the subgroup $O(n)$ in the group $Iso\ \mathbb{R}^n$, the group of motion (Cor 6.8 [3]).

Definition 3.1. A map $g : \mathbb{R}^n \to O(n)$ with $g(\mathbf{0}) = \mathbf{I_n}$ will be called a pseudo-deformation. The image $g(\mathbf{v})$ of $\mathbf{v}$ will be called pseudo-force corresponding to velocity $\mathbf{v}$. The corresponding operation $+_g$ on $\mathbb{R}^n$ given by $\mathbf{v} +_g \mathbf{w} = \mathbf{v}g(\mathbf{w}) + \mathbf{w}$, will be called pseudo-sum on $\mathbb{R}^n$.

Assume that every velocity $\mathbf{v}$ creates a field of force $g(\mathbf{v}) \in O(n)$. Then it determines a pseudo-deformation $g : \mathbb{R}^n \to O(n)$. Let Σ_1, Σ_2 be any two dynamical systems moving with velocities $\mathbf{v}$ and $\mathbf{w}$ respectively. Due to a pseudo-deformation g, the pseudo-sum $+_g$ on $\mathbb{R}^n$ will be $\mathbf{v}+_g\mathbf{w} = \mathbf{v}g(\mathbf{w})+\mathbf{w}$. In other words, we can say that the resultant of $\mathbf{v}$ and $\mathbf{w}$ under pseudo-force $g(\mathbf{w})$ will be $\mathbf{v}+_g\mathbf{w} = \mathbf{v}g(\mathbf{w})+\mathbf{w}$ instead of $\mathbf{v} + \mathbf{w}$. Thus, the relative velocity of Σ_1 with respect to Σ_2 will be $(\mathbf{v} +_g \mathbf{w}) - \mathbf{w} = \mathbf{v}g(\mathbf{w})$ instead of $\mathbf{v} + \mathbf{w} - \mathbf{w} = \mathbf{v}$. This suggests us to define the following:

Definition 3.2. Let $g : \mathbb{R}^n \to O(n)$ be a pseudo-deformation. Assume that Σ_1, Σ_2 be any two dynamical systems moving with velocities $\mathbf{v}$ and $\mathbf{w}$ respectively. Then the difference $(\mathbf{v} +_g \mathbf{w}) - \mathbf{w} = \mathbf{v}g(\mathbf{w})$ will be called effective velocity of Σ_1 under the pseudo-force $g(\mathbf{w})$. The integral $\int \mathbf{v}g(\mathbf{w})dt$ of effective velocity of Σ_1 will be called deformed-path of Σ_1 under the pseudo-force $g(\mathbf{w})$. It is denoted by $\mathbf{r}_d$. Thus, $\mathbf{r}_d = \int \mathbf{v}g(\mathbf{w})dt + \mathbf{c}$ and so the effective velocity will be $\frac{d\mathbf{r}_d}{dt} = \mathbf{v}g(\mathbf{w})$ under pseudo-force $g(\mathbf{w})$. The quantity $\frac{d^2\mathbf{r}_d}{dt^2}$ will be called effective acceleration of Σ_1 under a pseudo-force and the quantity $m\frac{d^2\mathbf{r}_d}{dt^2}$ will be called effective force acting on Σ_1 under a given pseudo-force, where 'm' is the mass of Σ_1.

Proposition 3.3. *Suppose that a particle of mass m is moving with velocity $\mathbf{q}$ in space whose path is $\mathbf{r} = \mathbf{r}(t)$, where t denotes the time. If there is a pseudo-force $\mathbf{A}(t) \in O(3)$ at time t. Then the deformed-path is given by*

$$\mathbf{r}_d = \int \frac{d\mathbf{r}}{dt}\mathbf{A}(t)dt + \mathbf{c} = \int \ (\mathbf{q}\mathbf{A}(t))\, dt + \mathbf{c}.$$

If $\mathbf{A}(t)$ is constant, say $\mathbf{A}$, throughout the motion, then the deformed path due to pseudo-deformation $\mathbf{A}$ is $\mathbf{r}_d = \mathbf{r}\mathbf{A} + \mathbf{c}$, $\mathbf{c}$ being the constant of integration.

Proof. Since the effective velocity of particle due to presence of pseudo-force $\mathbf{A}(t)$ at time t is $\mathbf{q}\mathbf{A}(t)$, where $\mathbf{q} = \frac{d\mathbf{r}}{dt}$. Thus, the deformed path $\mathbf{r}_d$ is

$$\mathbf{r}_d \quad = \quad \int \mathbf{q}\mathbf{A}(t)dt + vc$$

where $\mathbf{c}$ is the integrating constant. If pseudo-force $\mathbf{A}(t)$ is constant throughout the motion then the deformed path will be $\mathbf{r}_d = \mathbf{r}\mathbf{A} + \mathbf{c}$. □

From this it follows that the orbit of a satellite, planet etc., will be changed due to a pseudo-force. These pseudo-forces may exist due to asteroids, black holes, etc.

Proposition 3.4. *Suppose that a mass particle 'm' is moving in space $\mathbb{R}^3$ along a curve $\mathbf{r} = \mathbf{r}(t) \in \mathbb{R}^3$ under the action of force $\mathbf{F}$. If $\mathbf{A}(t)$ is a pseudo-force acting on the given mass particle. Then, its equation of motion is given by*

$$\mathbf{F}_e = \mathbf{F}\mathbf{A} + \mathbf{L}\frac{d\mathbf{A}}{dt}$$

where $\mathbf{L}$ *is the linear momentum of the mass particle and* $\mathbf{F} = m\ \frac{d^2\mathbf{r}}{dt^2}$.

Proof. Let P be a particle of mass m moving in space along a path $\mathbf{r} = \mathbf{r}(t)$. Let $\mathbf{F}$ be the force acting at P. Then, its equation of motion is given by

$$\mathbf{F} = m\frac{d^2\mathbf{r}}{dt^2} \tag{2}$$

Suppose that $\mathbf{A}(t) \in O(3)$ is a pseudo-force acting on the particle at time t. Then, its deformed path $\mathbf{r}_d$ is given by

$$\mathbf{r}_d = \int \frac{d\mathbf{r}}{dt}\mathbf{A}dt + \mathbf{c} \tag{3}$$

and so

$$\begin{aligned}\frac{d^2\mathbf{r}_d}{dt^2} &= \frac{d}{dt}\left(\frac{d\mathbf{r}}{dt}\mathbf{A}\right)\\ &= \frac{d^2\mathbf{r}}{dt^2}\mathbf{A} + \frac{d\mathbf{r}}{dt}\frac{d\mathbf{A}}{dt}\end{aligned}$$

Thus, the effective force $\mathbf{F}_e$ which causes the motion is given by

$$\begin{aligned}\mathbf{F}_e &= m\frac{d^2\mathbf{r}_d}{dt^2}\\ &= m\left(\frac{d^2\mathbf{r}}{dt^2}\mathbf{A} + \frac{d\mathbf{r}}{dt}\frac{d\mathbf{A}}{dt}\right)\\ &= \mathbf{FA} + \mathbf{L}\frac{d\mathbf{A}}{dt}\end{aligned}$$

where $\mathbf{L} = m\frac{d\mathbf{r}}{dt}$ is the linear momentum of the mass particle. □

Thus, the motion of a dynamical system will be affected due to the presence of pseudo-force. From the above, it follows that the magnitude $\mathbf{F}_e$ of force $\mathbf{F}_e$ at time t will be

$$\sqrt{\|\mathbf{F}\|^2 + 2\mathbf{FA}\left(\frac{d\mathbf{A}}{dt}\right)^T\mathbf{L}^T + \left\|\mathbf{L}\frac{d\mathbf{A}}{dt}\right\|^2}$$

where $\mathbf{A}^T$ denotes the transpose of $\mathbf{A}$.

If $\mathbf{A}(t)$ is independent of time, then $\frac{d\mathbf{A}}{dt} = \mathbf{0}$ and so the equation of motion under the pseudo-force $\mathbf{A}$ is given by $\mathbf{F}_e = \mathbf{FA}$. Thus, we have:

Corollary 3.5. *If* $\mathbf{A}$ *is a constant pseudo-force acting on the mass particle which is moving in space under the action of force* $\mathbf{F}$. *Then, its equation of motion is given by*

$$\mathbf{F}_e = \mathbf{FA}.$$

4. Streamlines under Pseudo-Deformation

Let $\mathbf{q} = (u, v, w)$ be a velocity of a blood particle at point $P(x, y, z)$. Due to electromagnetic field, suppose that the pseudo-force is $\mathbf{A} = [A_1, A_2, A_3] \in O(3)$, where A_1, A_2, A_3 are orthonormal column vectors in $\mathbb{R}^3$. Then, effective velocity of that blood particle at P will be $(\mathbf{q}A_1, \mathbf{q}A_2, \mathbf{q}A_3)$. Thus, the differential equation of streamlines:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

are changed into

$$\frac{dx}{\mathbf{q}A_1} = \frac{dy}{\mathbf{q}A_2} = \frac{dz}{\mathbf{q}A_3}$$

This shows that an electromagnetic field affects motion of blood particles. Thus, electromagnetic field affects the motion of blood particles and hence it will exert extra pressure on the heart. Also due to that field, the deformed motion causes tumors in effected area of our body.

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