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A Mathematical Model for Evaluating the Efficiency of the University of Kashan's Faculties

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Abstract

Efficiency evaluation of units has been of interest since many years in different domains such as management, economy, business, banking and many others. Data envelopment analysis is one of the popular operations research methods for measuring the relative efficiency of units, which use multiple inputs to produce multiple outputs. As we know, universities play a key role in many aspects of a country such as industry, economic, training and many others. Therefore, evaluating the efficiency of the faculties of a specific university is vital for effective allocation and utilization of educational resources, and consequently for enhancing its overall performance.

In this paper, we try to identify teaching and research strengths and weaknesses of each faculty of the University of Kashan and to provide a powerful tool for a fair comparison. To do this, we first determine the effective input and output variables for each teaching and research components. We then present a DEA model to evaluate both relative teaching and research efficiencies of each faculty of the University of Kashan.

Keywords: Data envelopment analysis, efficiency measurement, faculties' efficiency.

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1. Introduction

Daily developments of industry and increasing new demands have necessitated the relationship between knowledge and experience. Nowadays, relying on traditional

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industry is not sufficient, and we should use modern science to have more progress in the industry. Universities are the places that can provide this knowledge to the industry. In fact, universities are responsible to help industry by offering modern sciences. Nevertheless, training blindly and regardless of the results and its impact cannot be fruitful in the real world. Therefore, the issue of performance evaluation of universities should be seriously considered. In order to improve the performance of universities, it is necessary to evaluate efficiency of their faculties at least once a year. This evaluation should be based on scientific principles to provide reliable results. Generally, performance evaluation is one of the most difficult aspects of human management, so that some scholars have mentioned it as the vulnerable point of management. Moreover, performance evaluation is one of the effective management tools, which can help identifying strengths and weaknesses of the organization.

Due to the variety of activities and multiple objectives of the faculties, evaluation of their efficiencies is particularly complex. There are two main approaches for evaluating performance, parametric and non-parametric approaches.

The most common parametric method, which has been used for evaluating the efficiency of universities, is Stochastic Frontier Analysis (SFA) (see [12] for more details). In this approach, a specific functional form needs to be considered for the production function beforehand. For some reasons, SFA is not appropriate for evaluating the efficiency of academic units. First, the necessity of considering a specific function may be in conflict with the nature of units under evaluation. Second, it is not easy to apply this method in case of multi inputs and multi outputs. Third, computational complexity of parametric methods may be high by increasing the number of DMUs.

Data envelopment analysis (DEA), was introduced by Charnes et al. [7] and subsequently extended by Banker et al [3], is a methodology for assessing the relative efficiency of a set of homogeneous decision making units (DMUs), using several inputs to produce several outputs. DEA is one of the popular operations research methods. The popularity of DEA can be easily confirmed in the bibliography of Gattoufi et al [11] which lists more than 3000 previous contributions from 1978 to 2001. DEA is a popular approach for measuring the efficiency of non-profit institutions such as schools, hospitals and universities due to capability of multiple inputs and multiple outputs without a prior assumption on the monetary values of inputs and outputs.

There are several articles for evaluating efficiencies among universities and efficiencies among university departments or courses using DEA. Johnes et al [13], Beasly [4] and Stern et al [17] applied DEA technique to evaluate the university departments or courses, while some authors used DEA to evaluate universities, including Abbott et al [1], Avkiran [2], Johnes et al [14], Bougnol et al [5], Johnes et al [13], Breu et al [6] and Kuah et al [15].

Our goal in this research is to measure the efficiency of the faculties of Physics, Mathematical Sciences, Chemistry and Mechanical Engineering of the University of Kashan in both teaching and research perspectives during the academic year

2012-2013. For this purpose, we first determine the effective input and output variables for each teaching and research components, and then present a multi component DEA model, which is derived from Cook et al [8].

2. Mathematical Model

In this section, we first provide some preliminaries on DEA. Then, we describe the main mathematical model that is used in our application.

2.1. Basic DEA Model

The most basic DEA model is known as CCR model, which has been introduced by [7]. Suppose that there are n DMUs: $\text{DMU}_1, \ldots, \text{DMU}_n$. Each DMU_j $(j = 1, \ldots, n)$ uses m inputs x_{ij} $(i = 1 \ldots, m)$ to produce s outputs y_{rj} $(r = 1, \ldots, s)$. The efficiency of DMU_o, $(o = 1, \ldots, n)$, is obtained by solving the following linear programming, which is known as the multiplier form of CCR model in DEA.

$$
\begin{aligned}
e_o^* = \max \quad & \sum_{r=1}^{s} u_r y_{ro} && (1)\\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, \ldots, n, \\
& v_i \geq 0, i = 1, \ldots, m, \\
& u_r \geq 0, r = 1, \ldots, s,
\end{aligned}
$$

where the vectors v_i and u_r are the inputs and outputs weights, respectively. Model (1) is run n times to identify the relative efficiency scores of each DMU. Each DMU selects a set of v_i $(i = 1, \ldots, m)$ input weight and u_r $(r = 1, \ldots, s)$ output weight to maximize its efficiency score. The efficiency scores are between 0 and 1. Generally, DMU_o is efficient only if the optimal objective of model (1) equals to 1, i.e., $e_o^* = 1$; otherwise, it is inefficient. For more information about different topics of DEA approach, the interested readers can refer to the literature review done by Cook et al [9], Kuah et al [16], and the book written by Tone et al [10].

2.2. Description of the Main Model

For evaluating the teaching, research and overall efficiency of faculties, we use a model that was proposed by Cook et al in [8] in the different context. This model determines the overall efficiency of one DMU compared to all the other

DMUs when each DMU consists of two components. In fact, this model breaks the overall efficiency down into the components' efficiency. Cook et al [8] have proposed the following model for evaluating efficiency of multi-component DMUs,

$$
\begin{aligned}
e_o^a = \max \quad & u^1 Y_o^1 + u^2 Y_o^2 \\
\text{s.t.} \quad & v^1 X_o^1 + \bar{v}^{s_1} X_o^s + \bar{v}^{s_2} X_o^s + v^2 X_o^2 = 1, \\
& u^1 Y_k^1 + u^2 Y_k^2 - v^1 X_k^1 - \bar{v}^{s_1} X_k^s - \bar{v}^{s_2} X_k^s - v^2 X_k^2 \leq 0, \forall k, \\
& u^1 Y_k^1 - v^1 X_k^1 - \bar{v}^{s_1} X_k^s \leq 0, \forall k, \\
& u^2 Y_k^2 - \bar{v}^{s_2} X_k^s - v^2 X_k^2 \leq 0, \forall k, \\
& 0 \leq \alpha_i \leq 1, \forall i, \\
& (u^1, u^2) \in \Omega_1, (v^1, \bar{v}^{s_1}, \bar{v}^{s_2}, v^2) \in \bar{\Omega}_2, \\
& u_j^1, u_j^2, v_j^1, v_j^2 \geq \delta, \\
& \bar{v}^{s_1} \geq \alpha_i \delta, \bar{v}^{s_2} \geq (1-\alpha_i)\delta,
\end{aligned} \tag{2}
$$

where (X_k^i, Y_k^i), $i = 1, 2$ is the input-output vector of the ith component of DMU_k. The vector X_k^s indicates the shared inputs between two components of DMU_k, and the vector α is the proportionality variables. In addition, (u^1, v^1) and (u^2, v^2) indicate the input-output mix weights of components 1 and 2, respectively. Moreover, $\bar{v}^{s_1} = \alpha v^{s_1}$ and $\bar{v}^{s_2} = (1-\alpha)v^{s_2}$ where the vectors v^{s_1} and v^{s_2} indicate the multiplier weights for the shared components 1 and 2 resources, respectively. δ denotes the fact that an absolute lower bound δ may be in effect.

3. Data and Variables

We here identify the inputs and outputs variables of teaching and research components. Note that these inputs and outputs variables are identified based on previous studies and some measures that the authors believe to be essential in evaluating the efficiency of the university faculties.

3.1. Inputs and Outputs for Teaching Component

The point in teaching component is that universities hire professors to teach the enrolled students and produce graduates with high level of quality in terms of education. Teaching efficiency is therefore related to the performance of professors at universities in delivering knowledge to the students. Therefore, better professors will produce better quality products, in this case, the graduates; and consequently, scientific level of professors should be considered as an input measure. We also consider the quality of students as an input, and measure it based on the average of accepted ranks in the entrance exam. This is because that, better entry qualifications will produce better graduates. We consider the number of service courses presented by the other faculties as another input. This is because that, the professors of the other faculties help in producing teaching results. The outputs of teaching component are focused on graduates and educational results of each faculty, such as the overall average of each faculty, the average of the professors' assessment by students, and others. The inputs and outputs of teaching component are listed in Table 1.

Table 1: Inputs and outputs of teaching component.

Inputs	Outputs
x_1 : The average of students' qualification	y_1:The number of ranks acquired in the scientific matches
x_2 : The number of service courses presented by the other faculties	y_2: The ratio of the number of graduates to the whole number of students
x_3 : The number of students	y_3: The overall average of each faculty
x_4 : Scientific level of professors	y_4: The average of talented students
	y_5: The number of dissertations
	y_6: The percentage of non-conditioned students
	y_7: The percentage of A-averaged students
	y_8: The average of the professors' assessment by student

3.2. Inputs and Outputs for the Research Component

Another duty of university faculties is to produce research outputs such as the number of publications, inventions, published books and the extra-university projects. We consider the number of research staffs as an input variable. The research grant is also considered as an input, because more grants will produce better research outputs. Another research input is the scientific level of professors. This input is a common input in both teaching and research components. The inputs and outputs of research component are listed in Table 2.

Table 2: Inputs and outputs of research component.

Inputs	Outputs
x_4 : Scientific level of professors	y_9:The number of inventions
x_5 : Research grant	y_{10}: The number of publications
x_6 : The number of research staffs	y_{11}: The number of published books
	y_{12}: The extra-university projects

4. Implementing Model and Results

In this section, we measure the efficiency of Mathematical Sciences, Physics, Chemistry, and Mechanical Engineering faculties of the University of Kashan in both teaching and research perspectives. We first refine model (2) to evaluate teaching, research, and overall efficiencies of the University of Kashan's faculties. We then implement the refined model on the collected data for the academic year 2012-2013.

4.1. The Refined Model

As mentioned in the previous section, each faculty uses 4 inputs X_{ij} $(i = 1, 2, 3, 4)$ to produce 8 outputs Y_{rj} $(r = 1, \ldots, 8)$ from its teaching activities; and 3 inputs X_{ij} $(i = 4, 5, 6)$ to produce 4 outputs Y_{rj} $(r = 9, 10, 11, 12)$ from its research activities.

Note that input X_4 (Scientific level of professors) is common in both activities and therefore needs to be divided for the evaluation of teaching and research efficiencies. Since it is difficult to apportion the exact impact of scientific level of professor on research and teaching activities, the allocation for each faculty is done by maximizing its overall relative efficiency. Let α be the effectiveness of scientific level of professors in teaching activities and $1 - \alpha$ be the proportion of scientific level of professor on research activities. The following model is a simplified version of model (2) to evaluate the overall efficiency of the faculties,

$$
\begin{aligned}
E_o = \max \quad & U^1 Y_o^1 + U^2 Y_o^2 \\
\text{s.t.} \quad & V^1 X_o^1 + \bar{v}_4^1 x_{4o} + \bar{v}_4^2 x_{4o} + V^2 X_o^2 = 1, \\
& U^1 Y_k^1 + U^2 Y_k^2 - V^1 X_k^1 - \bar{v}_4^1 x_{4o} - \bar{v}_4^2 x_{4o} - V^2 X_k^2 \le 0, \forall k, k = 1, \ldots, 4, \\
& U^1 Y_k^1 - V^1 X_k^1 - \bar{v}_4^1 x_{4o} \le 0, \forall k, k = 1, \ldots, 4, \\
& U^2 Y_k^2 - V^2 X_k^2 - \bar{v}_4^2 x_{4o} \le 0, \forall k, k = 1, \ldots, 4, \\
& \frac{1}{3} \le \alpha \le \frac{1}{2}, \\
& U_j^1, U_j^2, V_i^1, V_i^2 \ge \epsilon, \forall i, j, \\
& \bar{v}_4^1 \ge \alpha\epsilon, \bar{v}_4^2 \ge (1 - \alpha)\epsilon,
\end{aligned} \tag{3}
$$

where $(U^1, V^1) = (u_1, \ldots, u_8, v_1, v_2, v_3)$ and $(U^2, V^2) = (u_9, \ldots, u_{12}, v_5, v_6)$ are the output/input mix weights of teaching and research components, respectively. The vectors $(X_j^1, Y_j^1) = (x_{1j}, x_{2j}, x_{3j}, y_{1j}, \ldots, y_{8j})$ and $(X_j^2, Y_j^2) = (x_{5j}, x_{6j}, y_{9j}, \ldots, y_{12j})$ are the inputs and outputs of teaching and research components of DMU_j, respectively. In addition, $\bar{v}_4^1 = \alpha v_4^1$ and $\bar{v}_4^2 = (1 - \alpha) v_4^2$ where v_4^1 and v_4^2 are the weights of the shared input, x_4, in teaching and research components, respectively. The teaching efficiency (TE_o) and research efficiency (RE_o) of DMU_o are defined as follows:

$$
TE_o = \frac{\sum_{r=1}^{8} u_r y_{ro}}{\sum_{i=1}^{3} v_i x_{io} + \alpha(v_4^1 x_{4o})} = \frac{\sum_{r=1}^{8} u_r y_{ro}}{\sum_{i=1}^{3} v_i x_{io} + \bar{v}_4^1 x_{4o}}, \tag{4}
$$

$$
RE_o = \frac{\sum_{r=9}^{12} u_r y_{ro}}{(1 - \alpha)(v_4^2 x_{4o}) + \sum_{i=5}^{6} v_i x_{io}} = \frac{\sum_{r=9}^{12} u_r y_{ro}}{\bar{v}_4^2 x_{4o} + \sum_{i=5}^{6} v_i x_{io}}. \tag{5}
$$

Model (3) is designed in such a way that it maximizes the overall efficiency (E_o) of DMU_o, as its objective function, subject to some constraints. Constraints 2, 3 and 4 of model (3) are to limit the relative overall efficiency, teaching efficiency and research efficiency of all DMUs to be within 1. Constraint 5 is to indicate the more effectiveness of scientific level of professors on the research activities compared to teaching; and to prevent zero proportion of the shared input on either activities. ϵ is a small non-Archimedean number, 0.002.

4.2. Results

Using model (3), we evaluate four faculties of the University of Kashan, Mathematical Sciences, Physics, Chemistry, and Mechanical Engineering during the academic year 2012-2013. The collected data are provided in Table 3 below. After solving model (3) using GAMS software, the relative teaching, research and overall efficiencies of the faculties are presented in Table 4.

Table 3: Training and research information.

Variables	Mathematical Sciences	Physics	Chemistry	Mechanical Engineering
x_1	5.1985	10.943	9.7739	26.2769
x_2	4	49	70	58
x_3	86	50	99	97
x_4	5.2	4.5	6.3	4.6
x_5	515383845	253845774	1014278300	349794500
x_6	55	49	106	109
y_1	1	0	0	0
y_2	0.24	0.28	0.35	0.23
y_3	13.25	13.90	14.69	14.29
y_4	17.16	16.60	16.66	16.92
y_5	27	31	63	9
y_6	0.24	0.28	0.10	0.15
y_7	0.70	0.71	0.53	0.53
y_8	0.845	0.848	0.851	0.870
y_9	0	0	2	0
y_{10}	27	31	63	9
y_{11}	0	1	1	1
y_{12}	0	4	5	3

It is worth mentioning that a faculty that is generally efficient, does not necessarily mean that it is efficient in both teaching and research components, for example see the efficiency scores of Chemistry faculty in Table 4. In fact, when the overall efficiency equals 1, it indicates that the faculty is efficient in producing outputs from its inputs. However, a teaching and research efficient faculty should generally be efficient, for example see the results for faculty of Physics.

As can be seen in Table 4, faculty of Physics is the only faculty that is both teaching and research efficient; and consequently overall efficient. Some faculties may be high in research efficiency but low in teaching. For example, faculties of Chemistry and Mechanical Engineering have better research performance than

Table 4: The teaching, research and overall efficiencies scores of faculties.

Faculties	Overall Efficiency	Training Efficiency(TE)	Research Efficiency(RE)
Mathematical Sciences	0.85	1	0.66
Physics	1.00	1.00	1.00
Chemistry	1.00	0.73	1.00
Mechanical Engineering	0.82	0.67	1.00

training. This indicates that they have more focus on research activities than teaching. On the other hand, due to teaching activities, faculty of Chemistry has generally better performance than Mechanical Engineering. In contrast, faculty of Mathematical Sciences has a better performance in terms of teaching as compared to research activities. It can be concluded that they have invested well on training, but unfortunately have no acceptable performance in terms of research activities.

5. Conclusion

This study proposes the Multi-component DEA approach for measuring the efficiencies scores of the university faculties. This paper applies an efficiency analysis to asses the efficiency of four faculties of the University of Kashan. We first introduce eight inputs and twelve outputs measures for each faculty. We then evaluate the efficiencies of faculties based on both their teaching and research activities. As future studies, some particular issues should be considered in evaluating the faculties of the University of Kashan; for example, considering manager's preferences on inputs and outputs measures, and providing some different effective solutions for enhancing the efficiencies of inefficient faculties with regard to their capacities.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this article.

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