

Laplacian Sum-Eccentricity Energy of a Graph

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Abstract

We introduce the Laplacian sum-eccentricity matrix \mathbf{LS}_e of a graph G , and its Laplacian sum-eccentricity energy $LS_eE = \sum_{i=1}^n |\eta_i|$, where $\eta_i = \zeta_i - \frac{2m}{n}$ and where $\zeta_1, \zeta_2, \dots, \zeta_n$ are the eigenvalues of \mathbf{LS}_e . Upper bounds for LS_eE are obtained. A graph is said to be twinenergetic if $\sum_{i=1}^n |\eta_i| = \sum_{i=1}^n |\zeta_i|$. Conditions for the existence of such graphs are established.

Keywords: Sum-eccentricity eigenvalues, sum-eccentricity energy, Laplacian sum-eccentricity matrix, Laplacian sum-eccentricity energy.

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1. Introduction

Let G be a simple connected graph with vertex set $\mathbf{V}(G)$ and edge set $\mathbf{E}(G)$, of order $|\mathbf{V}(G)| = n$ and size $|\mathbf{E}(G)| = m$. Let $\mathbf{A} = (a_{ij})$ be the adjacency matrix of G . The eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ of \mathbf{A} are the eigenvalues of the graph G [6]. Since \mathbf{A} is a symmetric matrix with zero trace, these eigenvalues are real with sum equal to zero. The energy of the graph G is defined as the sum of the absolute values of its eigenvalues [10, 16]:

$$E(G) = \sum_{i=1}^n |\lambda_i|.$$

After the introduction of the graph-energy concept in the 1970s [10], several other “graph energies” have been put forward and their mathematical properties extensively studied; for details see the recent monograph [12] and the survey [11].

In the last few years, a whole class of graph energies was conceived, based on the eigenvalues of matrices associated with a particular topological index. Thus, let TI be a topological index is of the form

$$TI = TI(G) = \sum_{v_i, v_j \in \mathbf{E}(G)} F(v_i, v_j)$$

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where F is a pertinently chosen function with the property $F(x, y) = F(y, x)$. Then a matrix \mathbf{TI} can be associated to TI , defined as

$$(\mathbf{TI})_{ij} = \begin{cases} F(v_i, v_j) & \text{if } v_i v_j \in \mathbf{E}(G) \\ 0 & \text{otherwise.} \end{cases}$$

If $\tau_1, \tau_2, \dots, \tau_n$ are the eigenvalues of the matrix \mathbf{TI} , then an “energy” can be defined as

$$E_{TI} = E_{TI}(G) = \sum_{i=1}^n |\tau_i|. \quad (1)$$

The most extensively studied such graph energy is the *Randić energy* [2,3,7,12], based on the eigenvalues of the Randić matrix \mathbf{R} , where

$$(\mathbf{R})_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_j}} & \text{if } v_i v_j \in \mathbf{E}(G) \\ 0 & \text{otherwise} \end{cases}$$

and where d_i is the degree of the i -th vertex of G . In an analogous manner the harmonic energy [14], *ABC energy* [9], geometric–arithmetic energy [23], Zagreb energy [15], and sum-eccentricity energy [22,26] were put forward.

To any energy E_{TI} of the form (1), a “Laplacian energy” LE_{TI} can be associated, defined as

$$LE_{TI} = LE_{TI}(G) = \sum_{i=1}^n \left| \theta_i - \frac{2m}{n} \right| \quad (2)$$

where $\theta_1, \theta_2, \dots, \theta_n$ are the eigenvalues of the matrix $\mathbf{LTI} = \mathbf{D} - \mathbf{TI}$, and where $\mathbf{D} = \mathbf{D}(G)$ is the diagonal matrix of vertex degrees.

The first such Laplacian energy, based on the adjacency matrix \mathbf{A} , was introduced in 2006 [13] and its theory is nowadays elaborated in full detail, see [12]. It is worth noting that this Laplacian energy found interesting engineering applications in image processing [18,25,27]. Bearing this in mind, it is purposeful to study other Laplacian graph energies. Some recent studies along these lines are [1,5,8,20,21].

In this paper we study the Laplacian version of the sum-eccentricity energy. In order to define it, we need some preparations.

The distance $d(u, v)$ between two vertices u and v in a (connected) graph G is the length of a shortest path connecting u and v [4]. The eccentricity of a vertex $v \in \mathbf{V}(G)$ is $e(v) = \max\{d(u, v) : u \in \mathbf{V}(G)\}$. The radius of G is $r(G) = \min\{e(v) : v \in \mathbf{V}(G)\}$, whereas the diameter of G is $d(G) = \max\{e(v) : v \in \mathbf{V}(G)\}$. Hence $r(G) \leq e(v) \leq d(G)$, for every $v \in \mathbf{V}(G)$.

In this paper, we denote by K_n , $K_{a,b}$, $K_{1,a}$, C_n , and P_n the complete graph, complete bipartite graph, star, cycle, and path, respectively.

The sum-eccentricity matrix of a graph G is denoted by $\mathbf{S}_e(G)$ and defined as $\mathbf{S}_e(G) = (s_{ij})$ [22, 26], where

$$s_{ij} = \begin{cases} e(v_i) + e(v_j) & \text{if } v_i v_j \in E \\ 0 & \text{otherwise.} \end{cases}$$

If $\mu_1, \mu_2, \dots, \mu_n$, are the eigenvalues of $\mathbf{S}_e(G)$, then the sum-eccentricity energy is

$$ES_e(G) = \sum_{i=1}^n |\mu_i|.$$

Definition 1.1. Let G be a graph of order n and size m . The Laplacian sum-eccentricity matrix of G , denoted by $\mathbf{LS}_e(G) = (\ell_{ij})$, is defined as

$$\mathbf{LS}_e(G) = \mathbf{D}(G) - \mathbf{S}_e(G).$$

The Laplacian sum-eccentricity spectrum of G , consisting of $\zeta_1, \zeta_2, \dots, \zeta_n$, is the spectrum of the Laplacian sum-eccentricity matrix. This leads us to define the Laplacian sum-eccentricity energy of a graph G as

$$LS_e(G) = \sum_{i=1}^n \left| \zeta_i - \frac{2m}{n} \right|. \quad (3)$$

If, in addition, we define the auxiliary quantity η_i as

$$\eta_i = \zeta_i - \frac{2m}{n}$$

then

$$LS_eE(G) = \sum_{i=1}^n |\eta_i|.$$

Lemma 1.2. Let G be an (n, m) -graph. Then

$$\sum_{i=1}^n \zeta_i = 2m.$$

Proof.

$$\sum_{i=1}^n \zeta_i = \text{trace}(\mathbf{LS}_e(G)) = \sum_{i=1}^n \ell_{ii} = \sum_{i=1}^n d_i = 2m.$$

□

Theorem 1.3. The Laplacian sum-eccentricity energy of the complete graph K_n is

$$LS_eE(K_n) = 4(n-1).$$

Proof. Recalling that the eccentricity of any vertex of K_n is unity, directly from the definition of the Laplacian sum-eccentricity matrix, we calculate that

$$\text{Spec}(\mathbf{LS}_e(K_n)) = \begin{bmatrix} -d & d+2 \\ 1 & n-1 \end{bmatrix}$$

where $d = n - 1$ is the degree of any vertex of K_n . Using the fact that $\frac{2m}{n} = n - 1 = d$, we get by Eq. (3)

$$\begin{aligned} LS_eE(K_n) &= | -d - d | + | d + 2 - d | + \cdots + | d + 2 - d | \\ &= 2d + 2(n - 1) = 4(n - 1). \end{aligned}$$

□

2. Bounds for Laplacian Sum-Eccentricity Energy

Theorem 2.1. *Let G be an (n, m) -graph. Then*

$$LS_eE(G) \leq \sqrt{n \left(\sum_{i=1}^n \sum_{j=1}^n \ell_{ij}^2 - \frac{4m^2}{n} \right)}. \quad (4)$$

Proof. We have

$$\begin{aligned} \sum_{i=1}^n \eta_i^2 &= \sum_{i=1}^n \left(\zeta_i - \frac{2m}{n} \right)^2 = \sum_{i=1}^n \left(\zeta_i^2 - \frac{4m}{n} \zeta_i + \frac{4m^2}{n^2} \right) \\ &= \sum_{i=1}^n \zeta_i^2 - \frac{4m}{n} \sum_{i=1}^n \zeta_i + \frac{4m^2}{n}. \end{aligned}$$

By Lemma 1.2,

$$\sum_{i=1}^n \eta_i^2 = \sum_{i=1}^n \zeta_i^2 - \frac{8m^2}{n} + \frac{4m^2}{n} = \sum_{i=1}^n \zeta_i^2 - \frac{4m^2}{n}.$$

Using the Cauchy–Schwarz inequality

$$\sum_{i=1}^n |\eta_i| \leq \sqrt{n \sum_{i=1}^n \eta_i^2}$$

we get

$$\sum_{i=1}^n |\eta_i| \leq \sqrt{n \left(\sum_{i=1}^n \zeta_i^2 - \frac{4m^2}{n} \right)}.$$

On the other hand,

$$\sum_{i=1}^n \zeta_i^2 = \text{trace}(\mathbf{LS}_e^2(G)) = \sum_{i=1}^n \sum_{j=1}^n \ell_{ij}^2$$

and inequality (4) follows. \square

It should be noted that inequality (4) is just a variant of the classical McClelland's upper bound for ordinary graph energy [16, 17].

Corollary 2.2. *Let G be an r -regular graph. Then*

$$\sum_{i=1}^n \eta_i^2 = \sum_{i=1}^n \sum_{j=1}^n \ell_{ij}^2 - nr^2.$$

Example 2.3. If $G \cong K_n$, then

$$\sum_{i=1}^n \eta_i^2 = 4n(n-1).$$

If $G \cong K_{a,b}$. Then

$$\sum_{i=1}^n \eta_i^2 = ab \left(a + b + 32 - \frac{4ab}{a+b} \right).$$

In particular, for $G \cong K_{1,a}$, with $n = a + 1$:

$$\sum_{i=1}^n \eta_i^2 = a^2 \left(1 - \frac{4}{a+1} \right) + 19a.$$

In what follows we derive another upper bound for the Laplacian sum-eccentricity energy using Weyl's inequality for matrices.

Theorem 2.4. (Weyl's inequality) [19] *Let \mathbf{X} and \mathbf{Y} be Hermitian $n \times n$ matrices. If for $1 \leq i \leq n$, $\lambda_i(\mathbf{X})$, $\lambda_i(\mathbf{Y})$, $\lambda_i(\mathbf{X} + \mathbf{Y})$ are the eigenvalues of \mathbf{X} , \mathbf{Y} , and $\mathbf{X} + \mathbf{Y}$, respectively, then*

$$\lambda_i(\mathbf{X}) + \lambda_n(\mathbf{Y}) \leq \lambda_i(\mathbf{X} + \mathbf{Y}) \leq \lambda_i(\mathbf{X}) + \lambda_1(\mathbf{Y}).$$

The matrices $\mathbf{LS}_e(G)$, $\mathbf{S}_e(G)$, and $\mathbf{D}(G)$ are all Hermitian $n \times n$ matrices. In addition, we use the facts that the eigenvalues of the diagonal matrix are the entries in the diagonal, and that the energy of a matrix \mathbf{X} is equal to the energy of $-\mathbf{X}$. We thus arrive at:

Theorem 2.5. *Let G be an (n, m) -graph with maximal vertex degree Δ . Then*

$$LS_e E(G) \leq ES_e(G) + k \Delta + \frac{2m}{n}(n-k) \quad (5)$$

where $k = |\{\zeta_i : \zeta_i \geq 2m/n\}|$.

Proof. Let $\zeta_1 \geq \zeta_2 \geq \dots \geq \zeta_n$ be the Laplacian sum-eccentricity eigenvalues, $\mu_1 \geq \mu_2 \geq \dots \geq \mu_n$ be the sum-eccentricity eigenvalues and $\rho_1 \geq \rho_2 \geq \dots \geq \rho_n$ be the eigenvalues of the degree matrix. We assume that $1 \leq k \leq r \leq n$. Using Theorem 2.4 we get

$$\mu_i + \rho_n \leq \zeta_i \leq \mu_i + \rho_1.$$

Since $\rho_n \geq 0$,

$$\mu_i \leq \zeta_i \leq \mu_i + \rho_1.$$

Since $2m/n \geq 0$,

$$\mu_i - \frac{2m}{n} \leq \zeta_i - \frac{2m}{n} \leq \mu_i + \rho_1.$$

Now we have to distinguish between two cases.

Case 1: If $\zeta_i - \frac{2m}{n} \geq 0$, then

$$\left| \zeta_i - \frac{2m}{n} \right| \leq \mu_i + \rho_1.$$

If there are k ζ_i 's, satisfy this condition, then

$$\sum_{i=1}^k \left| \zeta_i - \frac{2m}{n} \right| \leq \sum_{i=1}^k (\mu_i + \rho_1) \leq \sum_{i=1}^k |\mu_i| + k\rho_1. \quad (6)$$

Case 2: If $\zeta_i - \frac{2m}{n} \leq 0$, then

$$\left| \zeta_i - \frac{2m}{n} \right| \leq \left| \mu_i - \frac{2m}{n} \right|.$$

If we have, $\mu_i \leq 0$ for $i = k+1, \dots, r$ and $\mu_i \geq 0$ for $i = r+1, \dots, n$. Then

$$\begin{aligned} \sum_{i=k+1}^n \left| \zeta_i - \frac{2m}{n} \right| &\leq \sum_{i=k+1}^r \left| \mu_i - \frac{2m}{n} \right| + \sum_{i=r+1}^n \left| \mu_i - \frac{2m}{n} \right| \\ &= \sum_{i=k+1}^r \frac{2m}{n} + \sum_{i=k+1}^r |\mu_i| + \sum_{i=r+1}^n \frac{2m}{n} - \sum_{i=r+1}^n |\mu_i|. \end{aligned} \quad (7)$$

Combining the relations (6) and (7), we get

$$\begin{aligned} \sum_{i=1}^n \left| \zeta_i - \frac{2m}{n} \right| &\leq \sum_{i=1}^k |\mu_i| + k\rho_1 + \sum_{i=k+1}^r |\mu_i| - \sum_{i=r+1}^n |\mu_i| + \frac{2m}{n}(n-k) \\ &\leq ES_e(G) + k\rho_1 + \frac{2m}{n}(n-k) \end{aligned}$$

from which (5) follows straightforwardly. \square

Corollary 2.6. *If the graph G is r -regular, then*

$$LS_eE(G) \leq ES_e(G) + nr. \quad (8)$$

Proof. From Theorem 2.5, we have

$$LS_eE(G) \leq ES_e(G) + kr + \frac{2m}{n}(n - k).$$

Since, in addition, for an r -regular graph, $2m/n = r$,

$$LS_eE(G) \leq ES_e(G) + r(k + n - k)$$

and inequality (8) follows. \square

Lemma 2.7. [24] *For the complete bipartite graph $K_{a,b}$, the sum-eccentricity energy is $ES_e(K_{a,b}) = 8\sqrt{ab}$.*

Corollary 2.8. *For the complete bipartite graph $K_{a,b}$,*

$$LS_eE(K_{a,b}) \leq 8\sqrt{ab} + k \max\{a, b\} + \frac{2ab}{a+b}(a + b - k). \quad (9)$$

Proof. For $K_{a,b}$, $2m/n = 2ab/(a + b)$. Using Lemma 2.7, we get (9) from (5). \square

3. Twinenergetic Graphs

In this section, we point out a remarkable feature of Laplacian sum-eccentricity energy.

Definition 3.1. Let G be a graph of order n , and let ζ_i , $i = 1, 2, \dots, n$, be its Laplacian sum-eccentricity eigenvalues. We say that G is twinenergetic if

$$LS_eE(G) = \sum_{i=1}^n |\zeta_i|.$$

The above definition means that

$$\sum_{i=1}^n \left| \zeta_i - \frac{2m}{n} \right| = \sum_{i=1}^n |\zeta_i|. \quad (10)$$

The number of positive eigenvalues and negative eigenvalues (including their multiplicities) are denoted by $\zeta^+(G)$ and $\zeta^-(G)$, respectively. For the sake of simplicity, we assume that there are no zero Laplacian sum-eccentricity eigenvalues, i.e., that $\zeta^+(G) + \zeta^-(G) = n$.

Theorem 3.2. *A graph G is a Laplacian sum-eccentricity twinenergetic if it satisfies the following two conditions:*

$$\zeta_i(G) \geq \frac{2m}{n}, \quad i = 1, 2, \dots, \zeta_i^+(G). \quad (11)$$

$$\zeta^+(G) = \zeta^-(G). \quad (12)$$

Proof. Let $\zeta^+(G) = r$, where $1 \leq r \leq n$. Then

$$\begin{aligned} \sum_{i=1}^n \left| \zeta_i - \frac{2m}{n} \right| &= \sum_{i=1}^r \left| \zeta_i - \frac{2m}{n} \right| + \sum_{i=r+1}^n \left| \zeta_i - \frac{2m}{n} \right| \\ &= \sum_{i=1}^r \left| \zeta_i - \frac{2m}{n} \right| + \sum_{i=r+1}^n |\zeta_i| + \frac{2m}{n}(n-r). \end{aligned} \quad (13)$$

Let k be the number of eigenvalues satisfying the condition $\zeta_i(G) \geq \frac{2m}{n}$. Then, $1 \leq k \leq r$, and

$$\begin{aligned} \sum_{i=1}^r \left| \zeta_i - \frac{2m}{n} \right| &= \sum_{i=1}^k \left(\zeta_i - \frac{2m}{n} \right) + \sum_{i=k+1}^r \left(\frac{2m}{n} - \zeta_i \right) \\ &= \sum_{i=1}^k |\zeta_i| - \frac{2m}{n}k - \sum_{i=k+1}^r |\zeta_i| + \frac{2m}{n}(r-k). \end{aligned} \quad (14)$$

Substituting (14) back into (13) yields

$$\begin{aligned} \sum_{i=1}^n \left| \zeta_i - \frac{2m}{n} \right| &= \sum_{i=1}^k |\zeta_i| + \sum_{i=r+1}^n |\zeta_i| - \sum_{i=k+1}^r |\zeta_i| + \frac{2m}{n}(n-2k) \\ &= \sum_{i=1}^n |\zeta_i| - 2 \sum_{i=k+1}^r |\zeta_i| + \frac{2m}{n}(n-2k). \end{aligned}$$

If condition (11) is obeyed, i.e., if $k = r$, then

$$\sum_{i=k+1}^r |\zeta_i| = 0.$$

If, in addition, also condition (12) is obeyed, i.e., $2r = n$, then

$$\frac{2m}{n}(n-2k) = 0.$$

Thus, if both conditions (11) and (12) are satisfied, then the relation (10) holds, i.e., the graph G is twinenergetic. \square

It now remains to see if twinenergetic graphs exist at all. That such graphs do exist is verified by the following examples.

Example 3.3. The paths P_2 , P_4 , P_8 , and P_{16} are twinenergetic graphs.

For P_2 , direct calculations gives $\zeta_1 = 3$, $\zeta_2 = -1$, and $2m/n = 1$. Therefore,

$$\sum_{i=1}^2 |\zeta_i| = 4 \quad \text{and} \quad \sum_{i=1}^2 \left| \zeta_i - \frac{2m}{n} \right| = 4.$$

For P_4 we get $\zeta_1 = 9.0902$, $\zeta_2 = 4.7202$, $\zeta_3 = -2.0902$, $\zeta_4 = -5.7202$, and $2m/n = 6/4$. Therefore,

$$\sum_{i=1}^4 |\zeta_i| = \sum_{i=1}^4 \left| \zeta_i - \frac{2m}{n} \right| = 21.6208.$$

The cases P_8 and P_{16} are verified analogously.

Finding more examples of twinenergetic graphs, as well as their complete structural characterization remains a task for the future.

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