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1-Designs from the Group $PSL_2(59)$ and their Automorphism Groups

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Abstract

In this paper, we consider the projective special linear group $PSL_2(59)$ and construct some 1-designs by applying the Key-Moori method on $PSL_2(59)$. Moreover, we obtain parameters of these designs and their automorphism groups. It is shown that $PSL_2(59)$ and $PSL_2(59) : 2$ appear as the automorphism group of the constructed designs.

Keywords: Design, automorphism group, projective special linear group.

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1. Introduction

In [8], Key and Moori considered the primitive actions of the Janko groups J_1 and J_2 and construct some designs, codes and graphs. They proved that J_1 and J_2 apear as the automorphism group of thees combinatorial objects. Key et al. [10] applied the same method to the groups $PSP_n(q)$, $A_6 \cong PSL_2(9)$ and A_9 and their aim was to construct designs $\mathcal{D}$ from a group G such that $Aut(\mathcal{D})$ and $Aut(G)$ have no containment relationship. Motivated by the method used in [8, 9], Darafsheh et al. [5, 6] considered all the primitive actions of the groups $PSL_2(q)$, where $q = 8, 11, 13, 16, 17, 19, 23, 25, 27, 29, 31, 32,$ and found the parameters of the obtained designs and determined their automorphism groups. In following, Darafsheh et al. [7] considered the designs constructed by the action of the groups

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$PSL_2(q)$, where $q = 37, 41, 43, 47, 49$, and obtain parameters and automorphism groups for all the designs. Moreover, Darafsheh [4] considered the group $PSL_2(q)$, where q is a power of 2, and found a certain 1-design $\mathcal{D}$ invariant under the group $PSL_2(q)$ such that $Aut(\mathcal{D}) \cong S_{q+1}$.

In this paper, we consider the designs obtained by the primitive permutation representations of the group $PSL_2(59)$. Moreover, we obtain the automorphism groups of the constructed designs.

2. Preliminaries

Let p be a prime number, n be a positive integer and $q = p^n$. Denote by F_q the Galois field of order q. Let $GL_2(q)$ be the group of all the invertible 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ over the field F_q. Denote by $SL_2(q)$ the subgroup of $GL_2(q)$ consisting of the matrices with determinant 1. There is a natural action of $GL_2(q)$ on the 1-dimensional subspaces of the 2-dimensional vector space F_q^2. The kernel of this action is $N := \{\lambda I \mid 0 \neq \lambda \in F_q\}$. The projective general linear group $PGL_2(q)$ is the quotient $GL_2(q)/N$. The natural map

$$\begin{cases} \phi : & GL_2(q) \to F_q^*, \\ & \quad M \mapsto \det M, \end{cases}$$

is a group epimorphism with $\ker(\phi) = SL_2(q)$, where $F_q^* = F_q \setminus \{0\}$. The group $SL_2(q)$ also acts on the same set with the kernel $N \cap SL_2(q)$. Now, the projective special linear group $PSL_2(q)$ is defined to be $SL_2(q)/(N \cap SL_2(q))$. There is another approach for the definition of these linear groups. We add a distinguish symbol ∞ to F_q and associated to the invertible matrix $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ define the permutation

$$f_M(x) = \begin{cases} \frac{ax+b}{cx+d} & \text{if } x \in F_q \text{ and } cx + d \neq 0, \\ \infty & \text{if } x \in F_q \text{ and } cx + d = 0, \\ \frac{a}{c} & \text{if } x = \infty \text{ and } c \neq 0, \\ \infty & \text{if } x = \infty \text{ and } c = 0, \end{cases}$$

on $F_q \cup \{\infty\}$. The set of all such permutations is a subgroup of S_{q+1} isomorphic to $PGL_2(q)$ [12]. Moreover, a subgroup of $PGL_2(q)$ consists of those permutations f_M for which $ad - bc$ is a non-zero square in F_q, denoted by $L_2(q)$, is isomorphic to $PSL_2(q)$. In other words,

$$PSL_2(q) = \left\{ x \mapsto \frac{ax+b}{cx+d} \mid 0 \neq ad - bc \text{ is square} \right\}.$$

By [13], a maximal subgroup of $PSL_2(q)$ has one of the following shapes:

(i) A dihedral group of order $2(q-\epsilon)/d$, where $d = (2, q-1)$. Of course, exceptions occur when $\epsilon = 1$, $q = 3, 5, 7, 9, 11$ and $\epsilon = -1$, $q = 2, 7, 9$.

(ii) A solvable group of order $q(q-1)/d$.

(iii) A_4 when $q > 3$ is a prime number and $q \equiv 3, 13, 27, 37 \pmod{40}$.

(iv) S_4 when q is an odd prime number and $q \equiv \pm 1 \pmod 8$.

(v) A_5 when q is of one of the these forms: $q = 5^m$ or 4^m when m is a prime, q is a prime number congruent to $\pm 1 \pmod 5$, or q is the square of an odd prime number which satisfies $q \equiv -1 \pmod 5$.

(vi) $PSL(2, r)$ when $q = r^m$ and m is an odd prime number.

(vii) $PGL(2, r)$ when $q = r^2$.

For further properties of the groups $PGL_2(q)$ and $PSL_2(q)$, we refer the reader to [3, 13].

Let $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ be an incidence structure, where $\mathcal{P}$ and $\mathcal{B}$ are respectively point and block sets and $\mathcal{I}$ is a subset of $\mathcal{P} \times \mathcal{B}$. For any $p \in P$ and $B \in \mathcal{B}$, we write $p\ \mathcal{I}\ B$ if and only if $(p, B) \in \mathcal{I}$. We can replace the incidence relation $\mathcal{I}$ by the membership relation $\in$, i.e. the relation $(p, B) \in \mathcal{I}$ means $p \in B$. The incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ is called a $t-(\nu, k, \lambda)$ design if $|\mathcal{P}| = \nu$, $|B| = k$ for any $B \in \mathcal{B}$ and every t points of $\mathcal{P}$ is incident with exactly λ blocks of $\mathcal{B}$. The design $\mathcal{D}$ is called symmetric if the number of points is equal to the number of blocks, i.e. $\nu = b$. Let λ_s be the number of blocks through any set of s points, where $s \leq t$. We know that λ_s is independent of the set,

$$\lambda_s = \lambda \binom{\nu - s}{t - s} / \binom{k - s}{t - s}$$

and $\mathcal{D}$ is also an $s-(\nu, k, \lambda_s)$ design. A $t-(\nu, k, \lambda)$ design is called trivial if any subset of $\mathcal{P}$ with cardinality k is a block of $\mathcal{B}$. In this case, $b = \binom{\nu}{k}$. The dual of the incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ is $\mathcal{D}^t = (\mathcal{B}, \mathcal{P}, \mathcal{I})$. If $\mathcal{D}$ is a $t-(\nu, k, \lambda)$ design then $\mathcal{D}^t$ is a design with b points and the block size λ_1. The incidence matrix of $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ is a $|\mathcal{B}| \times |\mathcal{P}|$ matrix A with entries 0 or 1 whose rows and columns are labeled by blocks in $\mathcal{B}$ and points in $\mathcal{P}$ such that entry $(B, p) \in \mathcal{B} \times \mathcal{P}$ is 1 if and only if p is incidence with B. The incidence matrix of $\mathcal{D}^t$ is the transpose of A, A^t. Two structures $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ and $\mathcal{D}' = (\mathcal{P}', \mathcal{B}', \mathcal{I}')$ are called isomorphic and we write $\mathcal{D} \cong \mathcal{D}'$ if there is a one to one correspondence $\theta : \mathcal{P} \to \mathcal{P}'$ such that for all $p \in \mathcal{P}$ and for all $B \in \mathcal{B}$:

$$p\ \mathcal{I}\ B \longleftrightarrow \theta(p)\ \mathcal{I}'\ \theta(B).$$

The structure $\mathcal{D}$ is called self-dual if $\mathcal{D} \cong \mathcal{D}^t$. An isomorphism of $\mathcal{D}$ onto itself is said to be an automorphism of $\mathcal{D}$. The set of all the automorphisms of $\mathcal{D}$, denoted by $Aut(\mathcal{D})$, forms a group. It can be shown that if the incidence matrix of $\mathcal{D}$ is A then $Aut(\mathcal{D})$ consists of the pairs (P, Q) in which P and Q are permutation matrices on the rows and on the columns of A, respectively, and $PAQ = A$. For further properties of designs, see [1, 3].

3. Construction

Let G be a permutation group on a set Ω of size n. Suppose that B is a subset of Ω with $|B| \geq 2$. Then $\mathcal{D} = (\Omega, B^G, \in)$ is an incidence structure, where $B^G = \{B^g \mid g \in G\}$ forms the block set of $\mathcal{D}$ [3]. Moreover, we can see that if the action of G on Ω is t-homogeneous and $|B| \geq t$ then $\mathcal{D} = (\Omega, B^G, \in)$ is a $t-(\nu, k, \lambda)$ design with parameters $\nu = n$, $k = |B|$ and

$$\lambda = b\binom{k}{t}/\binom{\nu}{t} = \frac{|G|\binom{k}{t}}{|G_B|\binom{\nu}{t}},$$

where G_B denotes the stabilizer of B in G and $b = |B^G|$ is the number of all blocks of $\mathcal{D}$. In particular, when the action of G on Ω is transitive, we obtain a $1-(n, k, \lambda)$ design, where $\lambda = [G : G_B]k/n$. If the action of G on Ω is primitive and $B \neq \{\omega\}$ is an orbit of G_ω on Ω then $|G_B| = |G_\omega|$. In this case, the constructed design $\mathcal{D}$ has parameters $1-(n, k, k)$ and G acts on it as a group of automorphisms. The method we use in this paper is as follows:

Theorem 3.1. *[8, 9] Let G be a finite primitive permutation group acting on a set Ω of size n. Let $\alpha \in \Omega$ and $\{\alpha\} \neq \Delta$ be an orbit of the stabilizer G_α of α. If*

$$\mathcal{B} := \Delta^G = \{\Delta^g \mid g \in G\}$$

and

$$\mathcal{E} := \{\alpha, \delta\}^G = \big\{\{\alpha, \delta\}^g \mid g \in G\big\}$$

for a given $\delta \in \Delta$, then the incidence structure $\mathcal{D} = (\Omega, \mathcal{B})$ forms a symmetric $1-(n, |\Delta|, |\Delta|)$ design. Further, if Δ is a self-paired orbit of G_α then $\Gamma = (\Omega, \mathcal{E})$ is a regular connected graph of valency $|\Delta|$, $\mathcal{D}$ is self-dual and G acts as an automorphism group on each of these structures, primitive on vertices of the graph and on points and blocks of the design.

Therefore, our procedure for construction of designs according to the construction method outlined in Theorem 3.1 is as follows. Let G be a group and M be a maximal subgroup of G. Take Ω be the right cosets of M in G. Then, G acts primitively on the set Ω of size n. Choose $\omega \in \Omega$ and take Δ, where $|\Delta| = k > 1$, be an orbit of the stabilizer G_ω on Ω. By Theorem 3.1, Δ^G is the block set of a symmetric design $\mathcal{D}$ with parameters $1-(n, k, k)$. If the action of G on Ω is 2-transitive then G_ω, where $\omega \in \Omega$, has only two orbits ω and $\Omega \setminus \{\omega\}$ on Ω. Hence, the 1-design obtained in this way is trivial.

By [11], we can say that if $\mathcal{D}$ is a design constructed using the above method then

$$G \leq Aut(\mathcal{D}). \tag{1}$$

In [8], the authors conjectured that for any design $\mathcal{D}$ obtained from a primitive representation of a simple group G, we have $Aut(\mathcal{D}) = Aut(G)$. However, this conjecture is generally not true and $Aut(\mathcal{D})$ and $Aut(G)$ have no containment relation. In following, the authors [10] considered the simple groups A_6 and A_9. They found two designs $\mathcal{D}_1$ and $\mathcal{D}_2$ from the primitive permutation representations of A_6 and A_9, respectively, and showed that $Aut(A_6) \nleq Aut(\mathcal{D}_1)$ and $Aut(\mathcal{D}_2) \nleq Aut(A_9)$. So, it is interesting to construct designs from the primitive actions of a group G and then study the containment relation between the groups G, $Aut(G)$ and $Aut(\mathcal{D})$.

Table 1: 1-designs from the group $PSL_2(59)$.

No.	Max. Sub.	Degree	#	Length	$\lvert Aut(\mathcal{D})\rvert$
1	D_{58}	1770	46	29(28)	102660
				29(1)	205320
				58(2)	102660
				58(14)	205320
2	A_5	1711	38	6	102660
				10	102660
				12(2)	102660
				20(4)	102660
				30(5)	102660
				60(24)	102660
3	D_{60}	1711	45	15(2)	102660
				30(28)	102660
				60(14)	205320

4. Designs Constructed from the Group $PSL_2(59)$

Using Magma, we can see that the simple group $PSL_2(59)$ is a permutation group of order $102660 = 2^2 \times 3 \times 5 \times 29 \times 59$ generated by

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(3,54,48,16,5,10,46,23,26,51,30,28,27,38,59,21,50,56,19,49,45,52,34,
8,20,6,22,39,31)(4,25,36,35,33,12,37,40,17,53,58,47,15,9,60,32,24,41
,57,43,55,29,11,18,14,44,7,13,42)
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and

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(1,60,2)(3,31,59)(4,20,57)(5,42,58)(6,46,28)(7,26,17)(8,52,39)(9,30,
43)(10,54,23)(11,25,49)(12,44,27)(13,37,51)(14,29,22)(15,38,41)(16,5
6,34)(18,50,35)(19,32,53)(21,24,47)(33,48,40)(36,55,45)
```

acting on the set $\{1, 2, \ldots, 60\}$ of cardinality 60. Up to conjugacy, $PSL_2(59)$ has 5 maximal subgroups $M_1, M_2, \ldots, M_5$ of orders 58, 1711, 60, 60 and 60, respectively. By the shape of the maximal subgroups of $PSL_2(59)$, as noted in above, $M_1 \cong D_{58}$, $M_3 \cong M_5 \cong A_5$, $M_4 \cong D_{60}$ and M_2 is a solvable group. For the maximal subgroup M_2, the number of orbits of the stabilizer is 2 and so the action of $PSL_2(59)$ on the set of the cosets of M_2 is 2-transitive. Hence, the obtained design is trivial and we will not consider it. Moreover, Computations with Magma shows that the maximal subgroups M_3 and M_5 give us the same results. So, we set them in one row. By Magma, we see that the maximal subgroup M_i, where $i = 1, 3, 5$, is generated by the permutations α_i and β_i and moreover, M_4 is generated by α_4, β_4, γ_4 and δ_4 (See Appendix).

Table I contains all the information we obtain about the primitive representations of the group $PSL_2(59)$. In this table, the shapes of the maximal subgroups are given under the heading 'Max. Sub.'. The index of a maximal subgroup in $PSL_2(59)$ is given under the heading 'Degree' and the symbol '#' indicates the number of orbits of a point stabilizer in the action of $PSL_2(59)$ on the set of right cosets of a maximal subgroup. The word 'Length' denotes the length of the orbit of the stabilizer of a point and an entry $m(n)$ determines n orbits of length m. Also, the heading '$Aut(\mathcal{D})$' denotes the order of the automorphism group of the obtained design $\mathcal{D}$. All calculations have been carried out using Magma [2] (See Program in Appendix).

Theorem 4.1. *(i) The group $PSL_2(59)$ appears as the full automorphism group of some designs with parameters $1-(1770, 29, 29)$, $1-(1770, 58, 58)$, $1-(1711, 6, 6)$, $1-(1711, 10, 10)$, $1-(1711, 12, 12)$, $1-(1711, 15, 15)$, $1-(1711, 20, 20)$, $1-(1711, 30, 30)$ and $1-(1711, 60, 60)$.*

(ii) $Aut(PSL_2(59)) \cong PSL_2(59) : 2$ is the full automorphism group of some designs with parameters $1-(1770, 29, 29)$, $1-(1770, 58, 58)$ and $1-(1711, 60, 60)$.

Proof. By Theorem 3.1 and computations with Magma, we obtain the designs which are listed in above.

(i) If $\mathcal{D}$ is one of these designs then computations with Magma show that $|Aut(\mathcal{D})| = |PSL_2(59)|$. Now, inequality (1) implies that $Aut(\mathcal{D}) \cong PSL_2(59)$.

(ii) Computations with Magma show that the automorphism groups of these designs are isomorphic to each other. Denote by $\mathcal{D}$ the design $1-(1770, 29, 29)$. By Magma, $|Aut(\mathcal{D})| = 205320 = 2|PSL_2(59)|$ and there is a maximal subgroup N of $Aut(\mathcal{D})$ of index 2 such that $N \cong PSL_2(59)$. Moreover, we find the involution γ with the cycle type $2^{870}1^{30}$ in $Aut(\mathcal{D}) \setminus N$ (See Appendix). This implies that $Aut(\mathcal{D}) \cong PSL_2(59) : 2$. □

Appendix A: Generators

Generators of M_1:
α_1 =(1,20,29,51,41,46,31,4,10,6,52,49,38,33,60,8,45,37,9,7,28,27,21,
23,58,15,19,47,13)(2,50,16,44,48,5,40,42,36,35,56,54,26,18,55,3,30,2
5,14,11,57,53,59,32,17,22,12,34,43),
β_1 =(1,55)(2,60)(3,13)(4,42)(5,6)(7,17)(8,43)(9,22)(10,40)(11,58)(12
,37)(14,15)(16,38)(18,20)(19,25)(21,53)(23,57)(24,39)(26,29)(27,59)(
28,32)(30,47)(31,36)(33,50)(34,45)(35,46)(41,56)(44,49)(48,52)(51,54

).

Generators of M_3:
α_3 =(1,16,29)(2,48,55)(3,52,43)(4,57,54)(5,36,18)(6,30,60)(7,58,47)(
8,11,10)(9,13,51)(12,23,28)(14,45,32)(15,24,19)(17,33,25)(20,44,50)(
21,34,49)(22,27,38)(26,35,31)(37,41,59)(39,42,40)(46,56,53),
β_3 =(1,34)(2,54)(3,46)(4,15)(5,11)(6,42)(7,22)(8,56)(9,24)(10,60)(12
,38)(13,44)(14,39)(16,27)(17,52)(18,47)(19,53)(20,26)(21,31)(23,35)(
25,49)(28,45)(29,37)(30,57)(32,48)(33,51)(36,41)(40,58)(43,59)(50,55
).

Generators of M_4:
α_4 =(1,19,10,23,17)(2,54,36,33,55)(3,31,45,18,37)(4,6,5,11,49)(7,32,
30,59,20)(8,27,60,14,42)(9,51,43,50,12)(13,38,25,46,15)(16,21,57,58,
53)(22,35,26,44,28)(24,29,52,47,48)(34,40,39,41,56),
β_4 =(1,42,40)(2,48,30)(3,44,5)(4,18,35)(6,37,26)(7,33,52)(8,39,19)(9
,25,16)(10,27,41)(11,31,28)(12,38,53)(13,58,50)(14,34,17)(15,57,43)(
20,36,29)(21,51,46)(22,49,45)(23,60,56)(24,59,54)(32,55,47),
γ_4 =(1,4)(2,25)(3,56)(5,23)(6,17)(7,57)(8,22)(9,48)(10,11)(12,24)(13
,36)(14,26)(15,33)(16,30)(18,40)(19,49)(20,58)(21,32)(27,28)(29,50)(
31,41)(34,37)(35,42)(38,54)(39,45)(43,52)(44,60)(46,55)(47,51)(53,59
),
δ_4 =(1,58)(2,31)(3,55)(4,20)(5,32)(6,7)(8,12)(9,27)(10,16)(11,30)(13
,40)(14,43)(15,34)(17,57)(18,36)(19,53)(21,23)(22,24)(25,41)(26,52)(
28,48)(29,35)(33,37)(38,39)(42,50)(44,47)(45,54)(46,56)(49,59)(51,60
).

The involution in the proof of Theorem 4.1:
γ =(2,727)(3,246)(4,249)(5,161)(6,1120)(7,1378)(8,1015)(9,1479)(10,
1299)(11,801)(12,241)(13,1411)(14,406)(15,1190)(16,969)(17,847)(18,1
393)(19,1326)(20,1402)(21,1023)(22,646)(23,791)(24,1515)(25,1709)(26
,1238)(27,1022)(28,824)(29,72)(30,470)(31,466)(32,616)(33,500)(34,13
42)(35,666)(36,879)(37,1597)(38,1625)(39,1054)(40,1727)(41,563)(42,9
29)(43,469)(44,1460)(45,546)(46,549)(47,104)(48,1732)(49,685)(50,165
8)(52,262)(53,1103)(54,461)(55,208)(56,1406)(57,1735)(58,858)(59,116
8)(60,475)(61,1651)(62,1743)(63,602)(64,881)(65,479)(66,1229)(67,155
1)(68,1640)(69,1470)(70,353)(71,958)(73,1386)(74,823)(76,138)(77,106
9)(78,330)(79,1180)(80,1547)(81,181)(82,1064)(83,1720)(84,1596)(85,4
59)(86,1595)(87,1444)(88,1426)(89,900)(90,132)(91,827)(92,882)(93,21
9)(94,1391)(95,544)(96,923)(97,1095)(98,1279)(99,771)(100,633)(101,1
644)(102,1474)(103,513)(105,647)(106,167)(107,323)(108,1395)(109,948
)(110,977)(111,1153)(112,915)(113,885)(114,871)(115,1403)(116,787)(1
17,1016)(118,1249)(119,1174)(120,379)(121,282)(122,780)(123,1066)(12
4,732)(125,1572)(126,1198)(127,868)(128,944)(129,1700)(130,1433)(131

,213)(133,693)(134,1726)(135,1641)(136,564)(137,1609)(139,731)(140,1138)(141,1436)(142,186)(143,1080)(144,1121)(145,1139)(146,966)(147,1390)(148,1132)(149,830)(150,1274)(151,1005)(152,590)(153,1708)(154,1747)(155,445)(156,1696)(157,457)(158,1259)(159,1504)(160,1085)(162,440)(163,1226)(164,848)(165,1311)(166,1345)(168,331)(170,232)(171,1167)(172,1769)(173,220)(174,1337)(175,659)(177,1097)(178,342)(179,696)(180,797)(182,277)(183,1195)(184,1591)(185,1627)(187,1400)(188,533)(189,917)(190,1541)(191,652)(192,558)(193,1394)(194,1114)(195,442)(196,388)(198,887)(199,251)(200,1267)(201,261)(202,1420)(203,1661)(204,968)(205,1598)(206,1376)(207,807)(209,1216)(210,1296)(211,672)(212,1292)(214,400)(215,1177)(216,450)(217,1006)(218,964)(221,1610)(222,1300)(223,1191)(224,999)(225,846)(226,1492)(227,939)(228,1230)(229,813)(230,1029)(231,532)(233,1671)(234,391)(235,959)(236,1083)(237,1357)(238,957)(239,483)(240,1484)(242,1734)(243,1329)(244,896)(245,675)(247,779)(248,1046)(250,1466)(252,1570)(253,1048)(254,784)(255,748)(256,299)(257,1560)(258,583)(259,613)(260,1223)(263,1084)(264,1475)(265,1360)(266,737)(267,1423)(268,1227)(269,570)(270,452)(271,792)(272,399)(273,1464)(274,1204)(275,1392)(276,1284)(278,723)(279,1116)(280,1398)(281,1764)(283,430)(284,1527)(285,1129)(286,1721)(287,1090)(288,1707)(289,1030)(290,1742)(291,1762)(292,1594)(293,1257)(294,876)(295,859)(296,741)(297,754)(298,793)(300,1088)(301,795)(302,785)(303,1637)(304,606)(305,365)(306,683)(307,934)(308,1242)(309,360)(310,562)(311,1680)(312,1632)(313,1224)(314,1489)(315,425)(316,724)(317,1012)(318,1478)(319,719)(320,1607)(321,1636)(322,714)(324,869)(325,817)(326,1291)(327,701)(328,1178)(329,621)(332,1496)(333,1033)(334,1509)(335,516)(337,495)(338,1312)(339,1581)(340,662)(341,1487)(343,578)(344,482)(345,1037)(346,1372)(347,901)(348,663)(349,1352)(350,1381)(351,455)(352,503)(354,504)(355,1334)(356,1421)(357,519)(358,1497)(359,942)(361,574)(362,715)(363,1262)(364,971)(366,1653)(368,770)(369,677)(370,1036)(371,1500)(372,946)(373,1510)(375,1674)(376,1750)(377,1505)(378,1669)(380,760)(381,623)(382,640)(383,1156)(384,920)(385,1073)(386,1586)(387,1399)(389,975)(390,1086)(392,653)(393,1415)(394,1057)(395,909)(396,717)(397,523)(398,1072)(401,512)(402,821)(403,1457)(404,947)(405,840)(407,1265)(408,1447)(409,1622)(410,536)(411,1093)(412,1536)(413,1446)(414,1756)(415,1659)(416,658)(417,1210)(418,502)(419,1662)(420,540)(421,767)(422,1307)(423,1410)(424,845)(426,478)(427,1208)(428,1152)(429,718)(431,843)(432,894)(433,480)(434,1133)(435,1405)(436,589)(437,1059)(438,1159)(439,1443)(441,560)(443,1459)(444,1469)(446,1366)(447,1145)(448,1559)(449,553)(451,980)(453,1039)(454,1523)(456,1738)(458,508)(460,965)(462,1501)(463,905)(464,1716)(465,721)(467,638)(468,1162)(471,1522)(472,1441)(473,1712)(474,520)(476,674)(477,1170)(481,1571)(484,798)(485,1217)(486,1196)(487,1585)(488,862)(489,783)(490,1317)(491,691)(493,1428)(494,1266)(496,1493)(497,1203)(498,1011)(501,1235)(505,667)(506,620)(507,818)(509,1250)(510,1564)(511,7

20)(514,857)(515,832)(517,1091)(518,1553)(521,981)(522,1520)(524,106
3)(525,678)(526,1101)(527,888)(528,1549)(529,577)(530,571)(531,1276)
(534,1158)(535,1131)(537,1555)(538,1207)(539,1568)(541,1142)(542,118
6)(543,610)(545,781)(547,1336)(548,1060)(550,866)(551,1409)(552,806)
(554,810)(555,709)(556,1271)(557,1028)(559,751)(561,804)(565,1665)(5
66,1729)(567,1305)(568,1160)(569,856)(572,928)(573,916)(575,1695)(57
6,1058)(579,1166)(580,690)(581,1068)(582,1617)(584,1172)(585,1219)(5
86,989)(587,1321)(588,705)(591,1629)(592,1233)(593,1422)(595,1440)(5
96,1155)(597,1052)(598,632)(599,1455)(600,1135)(601,1358)(603,679)(6
04,1165)(605,1767)(607,742)(608,1140)(609,1437)(611,1567)(612,1642)(
614,1512)(615,733)(617,1181)(618,750)(619,1766)(622,1056)(624,1494)(
625,1346)(626,1745)(627,1445)(628,1272)(629,1503)(630,1741)(631,1539
)(634,699)(635,1467)(636,1290)(637,1269)(639,1710)(641,1508)(642,168
9)(643,951)(644,726)(645,878)(648,1450)(649,1468)(650,943)(651,1419)
(654,1343)(655,1119)(656,1107)(657,1179)(660,1098)(661,1245)(664,886
)(665,863)(668,1096)(669,960)(670,1614)(671,852)(673,1490)(676,1363)
(680,1740)(681,1253)(682,1718)(684,1306)(686,1588)(687,1349)(688,145
1)(689,1335)(692,1044)(694,1018)(695,1561)(697,1331)(698,765)(700,13
19)(702,790)(703,736)(704,1051)(706,1677)(707,762)(708,1759)(710,124
1)(711,1017)(712,1163)(713,839)(716,1731)(722,1148)(725,1643)(728,84
1)(729,1027)(730,1236)(734,1371)(735,1480)(738,1169)(739,1375)(740,1
228)(743,1295)(744,1438)(745,1200)(746,1184)(747,1298)(749,1502)(752
,1061)(753,1768)(755,1049)(756,1356)(757,899)(758,1546)(759,1461)(76
1,1255)(763,1576)(764,935)(768,1031)(769,1717)(772,931)(773,1328)(77
4,930)(775,1746)(776,1261)(777,1147)(778,956)(782,1013)(786,1540)(78
8,995)(789,1675)(794,1589)(796,1765)(799,1573)(800,1122)(802,1574)(8
03,1673)(805,1626)(808,1316)(809,1353)(811,1032)(812,962)(814,1694)(
815,1232)(816,835)(819,1202)(820,1215)(822,1664)(825,1737)(826,1618)
(828,854)(829,1722)(831,1344)(833,1020)(834,1369)(836,1130)(837,1035
)(838,1518)(842,1714)(844,1666)(849,1144)(850,1488)(851,1684)(853,16
06)(855,1634)(860,1024)(861,1189)(864,1511)(865,904)(867,1192)(870,1
432)(872,1624)(873,1218)(874,1099)(875,1600)(877,1014)(880,1670)(883
,1244)(884,1752)(890,974)(891,1686)(892,1280)(893,1263)(895,1118)(89
7,1693)(898,1477)(902,1538)(903,1185)(906,993)(907,1308)(908,1137)(9
10,1725)(911,937)(912,950)(913,1439)(914,1383)(918,1117)(919,1365)(9
21,1234)(922,1706)(924,1278)(925,992)(926,1367)(927,1289)(932,1333)(
933,1151)(936,1431)(938,1424)(940,1681)(941,1697)(949,1213)(952,1264
)(953,1348)(955,1486)(961,1283)(967,1417)(970,1507)(972,1691)(973,10
87)(976,1592)(978,1448)(979,1021)(982,1413)(983,1498)(984,1268)(985,
1055)(986,994)(987,1688)(988,1744)(990,1042)(991,1368)(996,1201)(997
,1483)(998,1338)(1000,1247)(1001,1542)(1002,1519)(1003,1652)(1004,11
27)(1007,1212)(1008,1733)(1009,1599)(1010,1273)(1026,1304)(1034,1309
)(1038,1141)(1040,1704)(1041,1323)(1043,1187)(1045,1350)(1047,1645)(
1053,1615)(1062,1416)(1065,1157)(1067,1286)(1070,1649)(1071,1715)(10

74,1577)(1075,1550)(1076,1373)(1077,1401)(1079,1602)(1081,1314)(1082
,1672)(1089,1513)(1092,1364)(1094,1556)(1100,1548)(1102,1260)(1104,1
563)(1105,1578)(1106,1407)(1108,1347)(1109,1531)(1110,1543)(1111,125
6)(1112,1297)(1113,1601)(1115,1638)(1123,1430)(1124,1654)(1125,1193)
(1126,1220)(1128,1339)(1134,1136)(1143,1657)(1146,1685)(1149,1302)(1
150,1425)(1154,1701)(1161,1387)(1164,1330)(1171,1569)(1173,1382)(117
5,1668)(1176,1667)(1182,1604)(1183,1499)(1188,1514)(1194,1751)(1197,
1427)(1205,1412)(1206,1318)(1209,1739)(1211,1285)(1214,1354)(1221,15
29)(1222,1524)(1225,1374)(1231,1239)(1237,1698)(1240,1753)(1243,1277
)(1246,1679)(1248,1517)(1251,1603)(1252,1404)(1254,1535)(1258,1647)(
1270,1320)(1275,1544)(1281,1293)(1282,1388)(1287,1770)(1288,1648)(12
94,1631)(1301,1408)(1303,1699)(1310,1471)(1313,1389)(1315,1476)(1322
,1532)(1324,1713)(1325,1453)(1327,1332)(1340,1613)(1341,1361)(1351,1
619)(1355,1528)(1359,1456)(1370,1758)(1377,1730)(1379,1396)(1380,139
7)(1384,1660)(1385,1458)(1414,1692)(1418,1566)(1429,1482)(1434,1516)
(1435,1485)(1442,1472)(1449,1575)(1452,1495)(1454,1633)(1463,1761)(1
465,1678)(1473,1705)(1481,1580)(1491,1552)(1506,1723)(1521,1749)(152
5,1663)(1526,1611)(1530,1703)(1533,1635)(1534,1650)(1537,1760)(1545,
1565)(1554,1719)(1557,1690)(1558,1755)(1579,1702)(1582,1605)(1584,16
21)(1590,1639)(1593,1763)(1612,1683)(1620,1623)(1628,1711)(1630,1754
)(1646,1656)(1655,1728)(1676,1757)(1682,1736)(1687,1724).

Appendinx B: A MAGMA Program

```
//The program, where g=PSL(2,59) and m is one of the its maximal
//subgroups
a1,a2,a3:=CosetAction(g,m);
st:=Stabilizer(a2,1);
orbs:=Orbits(st);
"no of orbits=",#orbs;
v:=Index(a2,st);
"degree",v;
lo:=[#orbs[j]:j in [1..#orbs]];
"seq of orbit length=",lo;
for j:=2 to #lo do
    "orbs no",j,"of length",#orbs[j];
    blox:=Setseq(orbs[j]^a2);
    des:=Design<1,v|blox>;
    autdes:=AutomorphismGroup(des);
    "aut des of order",Order(autdes);
    "---------------";
end for;
```

```
//omitting the trivial designs and the natural representations
```

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