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Mathematics, Music and Architecture

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Abstract

In simple terms, architecture and music are two very different things, which is the product of one set of materials in the form of one building and the other product is a set of sounds in the form of a song and melody. In this sense, architecture and music are two separate issues that are not similar. But with a little care in the hidden layer of music and architecture we can find amazing similarities. Finding the structural, quantitative and qualitative similarities between architecture and music is the focus of this article. Achieving a qualitative and quantitative correlation between these two arts can be a way to aesthetically improve architecture and achieve its healing principles. Therefore, the main purpose of this study is the understanding of the immediate beauty of music in mind and the application of its aesthetic elements in architecture. The research method is descriptive-analytical study of library documents and case studies. In this article, first, the definitions and the general structure of music and architecture are presented, then the results of the studies are introduced in the form of qualitative and quantitative adaptive tables. Finally, by analyzing data, the common structure of music and architecture is determined.

Keywords: Music, architecture, harmonic proportions, geometry.

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1. Introduction

In architecture, we have three spatial dimensions of length, width, and height, while the music formation background platform has only time dimension. By recognizing the elements of music and architecture and comparative study, they can

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find the similarity and common ground of music and architecture in qualitative issues such as hierarchy, rhythm, height, harmony, force, intensity, severity, stretching, peak, etc. In addition, the foundation of music and architecture is formed by mathematics and mathematical proportions. If the range of space that passes through the sound waves per unit time is a function of mathematical proportions, the ability to analyze and adapt little music and architecture is provided.

2. Research Background

Most of the research in the field of music and architecture has focused on the coherence of sensory and content aspects, and only a small amount has been given to the immersive repertoire of these two arts. Among the studies done in this area, the most significant work is related to Hesameddin Seraj (Iranian composer and artist, born in August 26, 1958, Isfahan, Iran), who shares musical and architectural interests [14]. He also noted the aesthetics of architecture and music [13]. Among other works in this field, we can mention the collection of articles presented at the first conference of architecture, music and literature of Iran. Also in the article comparative study of phonetic and visual in music and architecture this topic is briefly discussed [11]. Among the researchers from European and American countries, Iannis Xenakis and Le Corbusier [3] have been active in this field. Xenakis enjoyed the formalities of sound waves, and Le Corbusier's efforts ultimately led to a design based on musical notes, while the general form of cube and the rectangle has remained the same layout plan.

In the book "Point, Line, Level", in the translation of musical notes, Kandinsky (Wassily Wassilyevich Kandinsky, 16 December 1866 13 December 1944) has a way in which it can be perceived to some extent on the objective of the songs [16]. This method is in fact the translation of listening to the sight or song to the picture [14].

2.1 Aim

According to many philosophers such as Pythagoras (Ancient Greek philosopher and mathematician, circa 570 BC till 495 BC), Plato (The second philosopher of the great Greek philosophers, 423 BC till /347 BC), Al-Farabi (Muhammad ibn Muhammad Al Farabi, c. 872− between 14 December, 950 and 12 January, 951) and Avicenna (Ibn Sina or Abu Ali Sina c. 980− June 1037), there are two identical arts of music and architecture that use mathematics and geometry to convert quantity (sound and materials) to quality. Therefore, the main objective of this research is to study the comprehensive qualitative and quantitative musical and architectural communities in order to obtain the results of more efficient and accurate architectural design.

2.2 Research Methodology

The research method is a descriptive - analytical study of library documents and case studies. In this article, first the definitions and the general structure of music and architecture are presented, then the results of the studies are introduced in the form of qualitative and quantitative adaptive tables. Finally, by analyzing data, the common structure of music and architecture is determined. Two major questions on this topic are as follows:

1. What is the relationship between the elements of architecture and music?
2. Is it possible to enjoy the musical structure of the music to achieve spectacular and beautiful patterns in architecture?

3. Findings

musical proportions are closely related to art or architecture, and they can be considered as the basis of artistic designs [6]. The proportions in architecture are the relative and deductive relationships between the various components and all the elements [2]. In proportion to that of the one between the size of two things, we arrive at a certain ratio, which is said to be proportional to the ratio of these proportions. The fit can be obtained either visually or through mathematical calculations. Architectures proportion is very important because the architect, after choosing the type of building and the elements needed for it, must create an appropriate scale that takes into account each aspect of each section and the relationship between them at best [7]. In relation to the proportions and “Peymun”, used in the Islamic architecture of Iran, two categories are visible. The first category relates to the golden proportions $\sqrt{2}$ and $\sqrt{3}$ and the Iranian “Peymun”, which is determined on the basis of the numbers and proportions of the human body [5]. In the Islamic architecture of the “Peymun” system, there is direct relation with efficiency, suppleness and static.

By examining the proportions, we conclude that the objective architecture of geometry and music is an objective aspect of mathematics; therefore, the beauty of the two arts is the return to the beautiful proportions in geometry and number, which is in architecture with length, width and height, and in music with frequencies of notebooks. This consists of the platform of the architecture (location) and the context of the music. Wherever these proportions fit each other, the beauty of these two arts will be matched together.

3.1 Height and Navaak

In fact, the linear height is a linear quantity for expressing a point, a building, or a natural disorder, but more precisely it can be said: The height in the music range is the use of an audio or an orchestra but the height of the architecture is characterized by the length and shortness of the parts of a building and the proportions existing therein. More precisely, alertness (Navaak) or altitude is a perceptive quality of the sounds that allow them to grade and arrange, based on subtlety [11]. "Subtlety" as a mental phenomenon is equivalent to the changes that occur in the physical world for frequency of sounds. In common language, it is referred to as "Thin and Thick", and in older writings it has been referred to as "altitude". Usually, the sound of a higher frequency (such as a female voice) is often called "up", (soprano) and a lower frequency sound (like a male voice) "down", (Countertenor) [8].

3.2 Penumbra: The Bright and Shadow

The bright and shadow in music is called up and down, subtly, or loud, or short, or a musical effect. Each sound can be extracted from the lowest to the tiniest state with various instruments [11]. Shading in the traditional context of Yazd city, includes views and walls of the city with a desert background, which has a smooth skyline with gentle "tall and lofty".

This "tall" and "shortening" is more pronounced in Azerbaijani music and architecture. Azerbaijani instruments include wind instruments such as trumpets and trimmers, which are red in color, and a blue color in contrast to wind instruments such as "Persian Sorna", green and flute or straw [9].

3.3 Weight or Rhythm

"Rhythm" is a complex phenomenon that occurs in all works of art, a function of time in various visual-audio depictions, such as "the meaning of the word" in grammar. Rhythm is a phenomenon that does not exist and its existence is subject to another being. The existence of a rhythm in music is a fundamental concept and depends on the element of time.

One more musical work can be understood from its rhythm to its notes. The musician measures time as the architect makes up this feeling in the building's exterior [9]. The rhythm in music implies the regular repetition of time pieces, which is determined by the way in which music is detected (emphasis) [11]. In addition, it creates a sense of order while dynamic [9]. Rhythm and weight exist in time-dependent music and in space-related architecture, and there is no doubt that there is a certain harmony between the two [17].

3.4 Horizontal and Vertical Symmetry

Symmetry is one of the features that has been used extensively in the world of art and architecture, especially in the past. As shown in Figure 2 as an example, it also has many uses in music. The symmetry in the step by step will bring joy and pleasure, and if this symmetry collapses, it will cause sadness. The concept of geometric patterns is based on a number and its diagram in a world full of shapes and patterns. As shown in Figure 1 as an example, the traditional man understands these forms as the character of numbers in various aspects of the creator. This concept is numerically based on the symmetry and the conformance of the dimensions, shape and relative position of the components of a whole. In Islamic architecture, as well as the music and symmetry, in order to achieve perfection, there has been many uses [14].

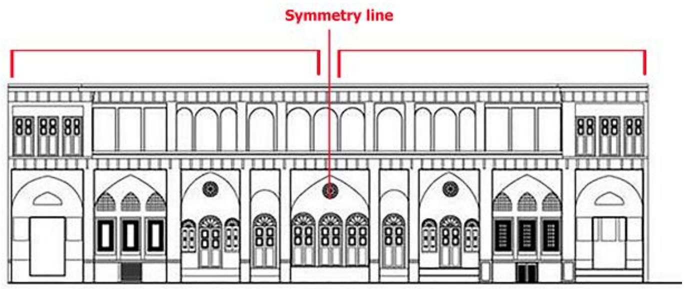

Figure 1: The symmetry in the east side of the main courtyard of the Ameri house in Kashan.

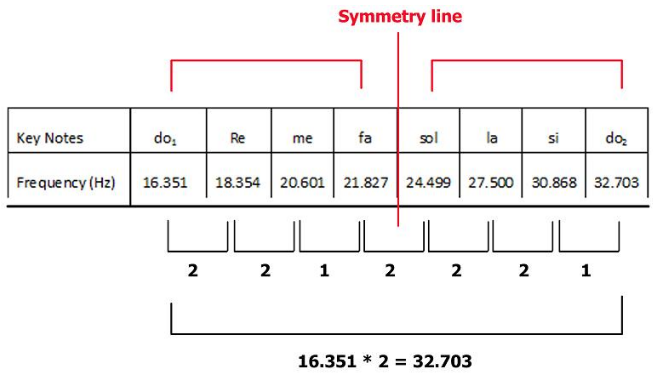

Figure 2: Symmetry line in major steps (Western) and “Mahoor Mode” (Iran).

3.5 Horizontal and Vertical Line

The set of points is the property of the movement and the motion towards the destination. The lines are horizontal, vertical, miles, curved, broken and spiral, each of which gives the viewer a special feeling. For example, the vertical line that goes up in architecture raises the point, is made in music by chords in the classical science and focuses on attention. The horizontal line in the architecture that brings about stability and relaxation in mind is expressed in music as a sound sentence and has the same effect on the audience. As shown in Figure 3 as an example, the soft and curved lines, as well as the miles of the lines that are in the architecture, can be seen in the notes of the piano [11].

In music notation, a tie is a curved line connecting the heads of two notes of the same pitch, indicating that they are to be played as a single note with a duration equal to the sum of the individual notes' values. A tie is similar in appearance to a slur, however slurs join notes of different pitches which need to be played independently, but seamlessly. Several notes in succession can be tied together. Such a succession can also be part of a larger, slurred phrase, in which case, ties and slurs must be used simultaneously and distinguishably [10].

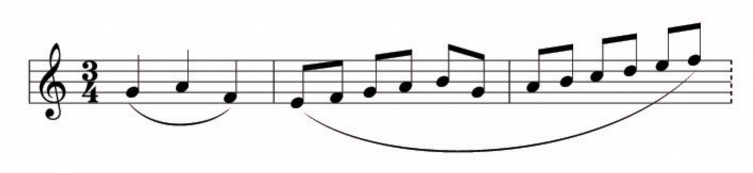

Figure 3: Line in the musical notes.

3.6 Hierarchical Order

In Chahargah mode (one of seven Iranian musical instruments: Mahoor, Shour, Homayoun, ...) from Azerbaijan, at first there is a withdrawal or beginning that will be the starting point of the music, and in fact we will understand the whole set of themes that runs through this device. In the traditional architecture, at the entrance to the building, we have an index entry that the architectural characteristics of this entry can be a sign of the style and genre of the whole building. Then as shown in Figure 4 and Table 1 as an example, you can enter into subsequent branches, respectively, and each of the branches is an architectural pattern: After the completion of each branch, there is a "rhythmic color" (Timbre, sometimes called "color" or "tone color" is the quality or sound of a voice or instrument) in one of the rhythms $(8.6, 4.3, 4.2)$ that can be found in urban architecture and design with the "collective" (market, square, center of the neighborhood and yard in the residential building) and movement (Passages, Porches and Footboards). Then again, "quiescence" or "rhythm (4.4)" can be meaningful in architecture as a "pause" space. The principle of hierarchy as one of the principles governing existence is most influential in the formation of confidentiality in the traditional architecture of urban architecture [12].

Table 1: Comparative study in the hierarchy of music and architecture.

Architecture	Music
$\Downarrow$	$\Downarrow$
Entrance	Entrance
$\Downarrow$	$\Downarrow$
Direction: Corridor	Branch: Opening Corner
$\Downarrow$	$\Downarrow$
Public Place: Central Coutryard	Rythmic Color: Wide Corner
$\Downarrow$	$\Downarrow$
Pause Space	Silence or Rythm 4,4
$\Downarrow$	$\Downarrow$
Public Place: Caravanserai	Rythmic Color: Iraq Corner

(a) Iraq Corner

(b) Wide Corner

(c) Opening Corner

(d) Entrance

Figure 4: Hierarchy in music and architecture.

3.7 Musical Modal System in Architecture

The basic step in the music known as the "Major" step and in Iranian music is near the "Mahoor" step. By examining these steps, the geometric ratios, their harmony and their arithmetic are determined. These proportions make it look beautiful. For example, enjoying a mathematical fit in the major musical steps in classical European music (especially in the D Major step, which is there to show a sense of conquest, happiness and cheerfulness) and Mahoor in Iranian musical modal system due to adaptation. On the natural step, it is based on a balanced state; it is used to express the epic state of internal strength [14]. In contrast, to express the sensation of softness and friendship, minority music steps (especially the E Minor step) are used to demonstrate the extreme fear and anxiety of the $D\sharp$ Minor step in architecture. For example, the architectural proportions of the Khaju Bridge in Isfahan, which is based on the mathematical proportions of Step Mahoor and Caravanserai Maghsoud Beyk, based on the mathematical proportions of the Shour step(one of seven Iranian musical modal system) in Iranian music, can be noted [4]. The Shour step is a small (minor) fashion, since the third degree (in the image, the thirty blasts) and the sixth (middle) and the seventh grade (Fa), which are the degrees of the medal, are comparable to the "tuberculosis". The base note is a small distance (similar to a small western step); the distance between these degrees corresponds to the degree of quadruple curvature in the second degree, with the degrees of the Greek modes or the small theoretical step (the theoretical Minor) [4].

3.8 Arithmetic, Geometric and Harmonic Proportions

Sound is an instrument by which temporary patterns of frequency can be converted into geometric and spatial patterns. Music is the science of sounds and art is decorated and combined and supplied in such a way that it is comfortable for the human ear. Harmonics are audio sounds that are always accompanied by a main sound and their frequencies (Hz) are the correct number of original sound frequencies. The beauty or ugliness of the sounds, the melodious or unpleasant sounds of them is related to harmonics. The more frequent the harmonics and their frequencies are, the more original, beautiful and more interesting sounds will be, and vice versa. The sounds used in Iranian music are said Naghmeh to be the equivalent of the same Note in European music. Naghmeh or Notes include the seven main Notes that have different titles in different countries, but in France they are named by do, re, me, Fa, sol, la, si. The proportions of audio beauty in the music world are based on proportions that are inside music. The beauties are the result of consistent and harmonious proportions, which, in obvious proportions such as latitude, longitude and height, and in hidden proportions, such as optical and acoustic vibrations, are the constructors of visual and audible beauties. So the reason for the beauty in art and architecture is the return to beautiful proportions in geometry and numbers.

3.8.1 Proportions in the Frequency of the Notes

Mathematical proportions in architecture can be seen in the ratio of length, width and height of the volume of the building or the ratio of openings (in the floor, wall and ceiling). As shown in Table 2 as an example, in music, these ratios are expressed by the frequency of the notes. Wherever these proportions fit each other, the beauty of these two arts will be matched together.

Table 2: Music frequency in Major steps.

Key Notes	do_1	re	me	fa	sol	la	si	do_2
Frequency (Hertz)	16.351	18.354	20.601	21.827	24.499	27.500	30.868	32.703

As shown in Figure 5, if we assume the basis of the step is “do”, the first sound that creates a good sound is basically “fa”. This ratio is one of the very common ratios of forms in architecture, painting and photography.

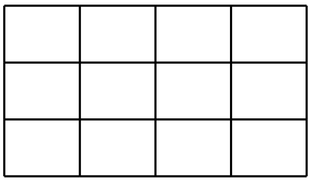

Figure 5: Geometric representation of 3 : 4 ratio.

$$\frac{do}{fa} = \frac{16.35Hz}{21.82Hz} = \frac{3}{4} = 0.75.$$

As shown in Figure 6, the second sound is “sol”, decent, which is another common aspect in the visualization of forms. The distance between “do” and “sol” is the most basic tune in the eastern and western instruments.

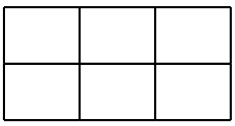

Figure 6: Geometric representation of 2 : 3 ratio.

$$\frac{do}{sol} = \frac{16.35Hz}{24.50Hz} = \frac{2}{3} = 0.66.$$

As shown in Figures 5 and 6, the "do-fa" and "do-sol" intervals saturate the mind in succession and are simultaneously "harmonics". As specified in Figure 7 the third voice is aligned with "do1", in the next step "do2", or octave sound.

Figure 7: Geometric representation of 1 : 2 ratio.

$$\frac{do1}{do2} = \frac{16.35Hz}{32.70Hz} = \frac{1}{2} = 0.5.$$

Between the two steps, "do1" and "do2" have infinite sound, but the sounds are musical, which together create delightful intervals, or, in other words, they are persuasive and pleasing when played continuously. These intervals always have special mathematical proportions that can be generalized in other arts. The fourth and fifth distances, namely, "fa / do" and "sol / do", are the most reliable distances that are common in all steps because they are normal [14]. From the composition of the notes, there are relatively short sentences that are pleasant to hear. A music sentence may be very beautiful, but not enough. It is not enough to express a sentence alone to express and convey emotions, and it is necessary to combine sentences, speeches, and essays to be useful to the listener and to call it "piece" in musical language. A tune, song, melody, a relatively short piece of distance between the notes and the beat, and the one that is identical and identifiable to the listener [14]. If architecture is considered to be the application of proportions in spatial volume, then music can be considered as the application of proportions over time. The ratio of frequencies to each other carries a pleasant or unpleasant effect on humans. Therefore, the study of the relatives, which are the factors creating beauty, is necessary.

In music, the addition of harmonic downward harmonics and the calculus give two extreme repeated double octave proportions, which yields 1, 3/4, 2/3, 2, and correlates with the corresponding music. In other words, the average of the calculus and the harmony between the two-dimensional geometric ratios is numerical ratios that relates to the full-fledged and full-fledged distances of the fourth, that is to say, to the extent that the fundamental voices in all the musical steps correspond. Among the two numbers, 1 and 2 are $\sqrt{2}$, which is very popular in Iranian music and architecture. This number is used in Iranian tile and has a golden fit. The complementary optical frequencies that are harmonic and contrast coordinates also have the same proportions [15].

3.8.2 Parameters in Harmonic Series

To understand how to translate the musical architecture, first, it is necessary to get acquainted with a variety of arithmetic, geometric, harmonic proportions, and then to the fundamental concepts of music. An appropriate fit is a set of triangular numbers in which (c) is greater than (a) and (b). The relationship between these three numbers is presented as a formula.

1. If the proposed formula is $b = \frac{a+c}{2}$, then the proportion is called “arithmetic”.
2. If the formula presented is $b = \sqrt{ac}$, then the fit is called “geometric”.
3. If the proposed formula is $b = \frac{2ac}{(a+c)}$, then the proportion is called “harmonic”.

An important and conspicuous feature of harmonic fit is that the image of each harmonic progression is an arithmetic progression. So:

An upward arranged progression: 1, 2, 3, 4, 5, etc.

A descending progression: 1.1, 1.2, 1.3, 1.4, 1.5, etc.

As shown in Figure 8, this seriesis called the harmonic series:

$$\sum_{k=1}^{\infty} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots .$$

How?

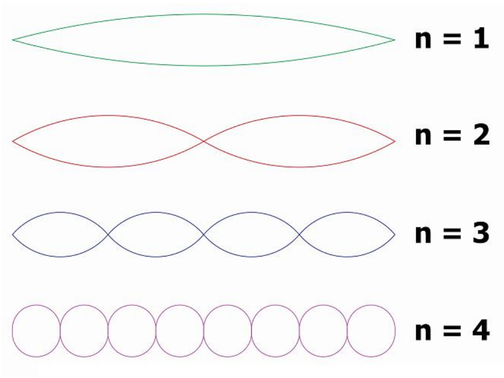

Figure 8: Harmonic series.

If $a = \frac{1}{1}$ and $c = \frac{1}{3}$, then

$$b = \frac{2ac}{a+c} \qquad b = \frac{2 \times 1 \times \frac{1}{3}}{1 + \frac{1}{3}} \qquad b = \frac{1}{2}$$

and if $a = \frac{1}{2}$, $c = \frac{1}{4}$, then

$$b = \frac{2ac}{a+c} \qquad b = \frac{2 \times \frac{1}{2} \times \frac{1}{4}}{\frac{1}{2} + \frac{1}{4}} = \frac{1}{3} \qquad b = \frac{1}{3}$$

If $a = \frac{1}{3}$, $c = \frac{1}{5}$, then

$$b = \frac{2ac}{a+c} \qquad b = \frac{2 \times \frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} + \frac{1}{5}} \qquad b = \frac{1}{4}$$

... and etc.

3.8.3 Harmonic Proportions in Geometry

Therefore the harmonic series can be computed with the help of the harmonic proportion formula and match them. But how can harmonic ratios be used in other arts, and especially in architecture? As shown in the Figure 9 the intersection of the trapezoidal diameters of two similar triangles is created.

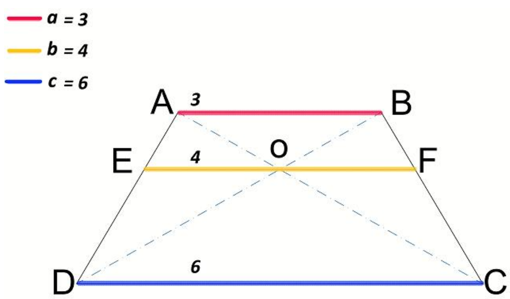

Figure 9: Geometric definition of harmonic ratio.

In two similar triangles ABO and DOC:

$$\frac{AB}{DC} = \frac{OB}{OD} = \frac{OA}{OC} = \frac{3}{6} = \frac{1}{2} \quad \Rightarrow DO = 2OB.$$

Also in triangle BCD:

$$\frac{OF}{DC} = \frac{OB}{DB} = \frac{OB}{OD + OB}$$

$$\begin{aligned}\frac{OF}{DC} &= \frac{OB}{\frac{(DC\times OB)}{AB}+OB}\\ &= \frac{OB}{\frac{(DC\times OB)}{AB}+\frac{(AB\times OB)}{AB}}\\ &= \frac{OB}{\frac{OB\times(DC+AB)}{AB}}\\ &= \frac{OB\times AB}{OB\times(DC+AB)}\\ &= \frac{AB}{DC+AB}.\end{aligned}$$

$$\begin{aligned}\frac{OF}{DC} &= \frac{AB}{DC+AB}\\ OF &= \frac{DC\times AB}{DC+AB}\\ 2OF &= EF=\frac{2(DC\times AB)}{DC+AB}\\ b &= \frac{2ac}{c+a}.\end{aligned}$$

In this way, the harmonic equation formula is proved in this geometric form. For the exam, we put the length of the trapezoid once in the harmonic equation formula to obtain the length of the EF based on it:

If $a=3$, $c=6$,

$$b=\frac{2ac}{c+a}\qquad b=\frac{[2\times 3\times 6]}{3+6}=\frac{36}{9}\qquad b=4.$$

As proved, this is one of the geometric manifestations of harmonic proportions. As shown in Figure 10 as an example, this formulation and this geometric form are very much used in the perspective of architecture. In drawing of one point perspective, finding the intersection of the diameters is one of the main methods; and again we have: $b=\frac{2ac}{c+a}$.

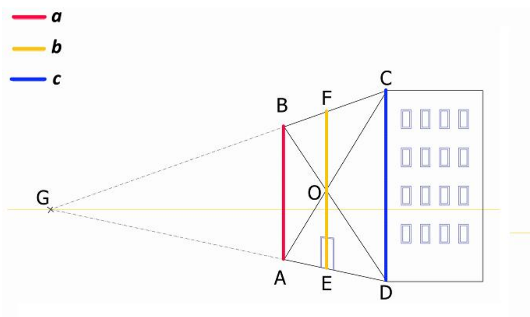

Figure 10: Gaining the middle of the wall in a point-by-point perspective.

3.8.4 Harmonic Proportions in Case Studies

But has this formula been used in the construction of architectural monuments? To come up with the answer to this question, let's look at a case study.

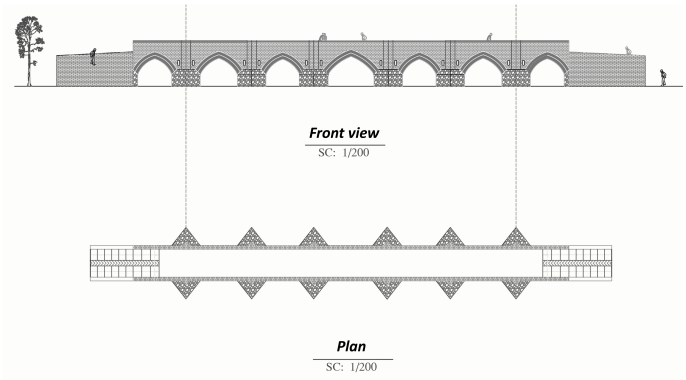

Figure 11: Yeddigozn (7 eyes) bridge in Ardabil.

The Bridge of Yeddi Goz has seven springs with stone basements and bricks. This bridge has a symmetrical position and the middle Mouth, causing the bridge to be angled. This bridge was built during during Safavid rule. Since 1995 its repair operation has begun. As shown in Figure 11, the bridge consists of seven brick openings with arches of five and seven and rock breakwaters. The mortar is used for building bridges includes plaster and lime. This bridge is located on the Saheli Street on the Baliglu river and it provides passing of light vehicles and pedestrians.

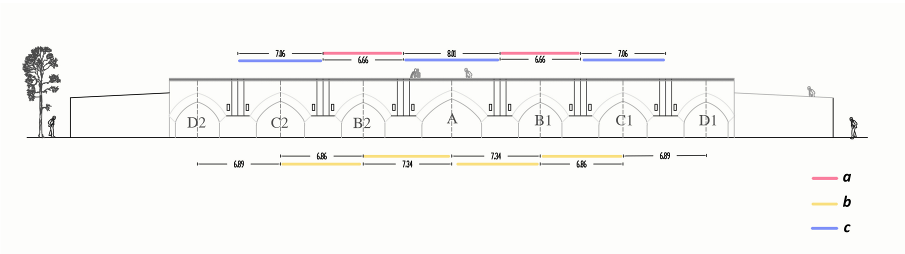

Figure 12: Harmonic proportions in Yeddigoz (7 eyes) bridge in Ardabil.

As shown in Figure 12, in the craters $(A - B1)$ and $(A - B2)$ of the bridge; a and c is the $x-$axis to the axes of the bridge and b is the distance from the arc to the next arc.

$$a = 6.66m \qquad c = 8.01m,$$

and based on the formula: $b = \frac{2ac}{c+a}$

$$b = \frac{2 \times 6.66 \times 8.01}{6.66 + 8.01} = \frac{106.6932}{14.67} = 7.2728m.$$

The actual dimensions of b are $7.34m$, which is very close to the extracted number in the formula. Also in the craters $(B1 - C1)$ and $(B2 - C2)$ of the bridge:

$$a = 6.66m \qquad c = 7.06m$$

and based on the formula: $b = \frac{2ac}{c+a}$

$$b = \frac{2 \times 6.66 \times 7.06}{6.66 + 7.06} = \frac{94.039}{13.72} = 6.857m.$$

The actual dimensions of b are $6.86m$, which is very close to the extracted number in the formula.

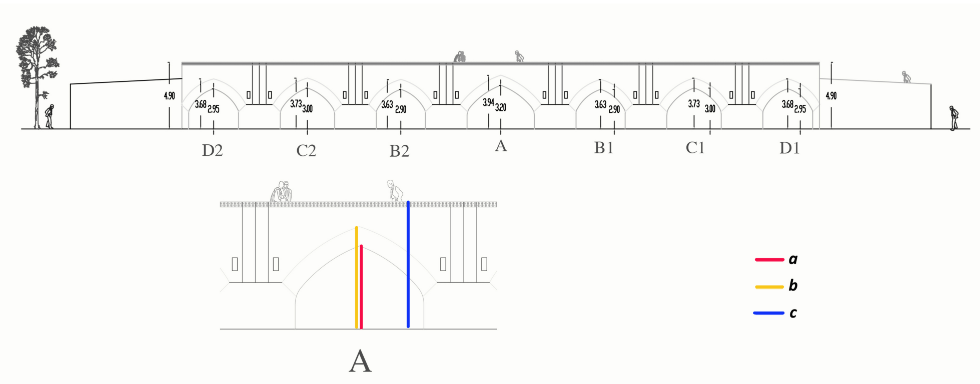

Figure 13: Harmonic proportions in Yeddigoz (7 eyes) bridge in Ardabil.

As shown in Figure 13, in the crater (A) of the bridge; a is the interior height of the arc, b is the outer height of the arc and c is the height of the bridge to the edge of the wall.

$$a = 3.20m \qquad c = 4.90m$$

and based on the formula: $b = \frac{2ac}{c+a}$

$$b = \frac{2 \times 3.2 \times 4.9}{3.2 + 4.9} = \frac{31.36}{8.1} = 3.87m.$$

The actual dimensions of b are $3.94m$, which is very close to the extracted number in the formula. Also in the craters $(B1)$ and $(B2)$ of the bridge:

$$a = 2.90m \qquad c = 4.90m$$

and based on the formula: $b = \frac{2ac}{c+a}$

$$b = \frac{2 \times 2.9 \times 4.9}{2.9 + 4.9} = \frac{28.42}{7.8} = 3.64m.$$

The actual dimensions of b are $3.63m$, which is very close to the extracted number in the formula. Also in the craters ($C1$) and ($C2$) of the bridge:

$$a = 3.00m \qquad c = 4.90m$$

and based on the formula: $b = \frac{2ac}{c+a}$

$$b = \frac{2 \times 3 \times 4.9}{3 + 4.9} = \frac{29.4}{7.9} = 3.72m.$$

The actual dimensions of b are $3.73m$, which is very close to the extracted number in the formula. Also in the craters ($D1$) and ($D2$) of the bridge:

$$a = 2.95m \qquad c = 4.90m$$

and based on the formula: $b = \frac{2ac}{c+a}$

$$b = \frac{2 \times 2.95 \times 4.9}{2.95 + 4.9} = \frac{28.91}{7.85} = 3.68m.$$

The actual dimensions of b are $3.68m$, which is exactly equal to this number in the formula. In this bridge, there are interesting examples of Fibonacci number, which there is no direct connection to the discussion of this article, are ignored.

3.8.5 Harmonic Proportions in Drawing

As shown in the case study, harmonic proportions are widely used in the design of this same bridge in traditional Iranian architecture. But using this formula in architectural drawings seems a bit complicated. Is there a simpler way to reach these proportions when designing? Fortunately, there are some ways to achieve this, some of which are here.

- The first method:

  If we have three segments called a, b, c that is: $a < b < c$, There are harmonic proportions between them and only length c is specified. We use this geometric method to find the length of the lines a and b:

  As shown in Figure 14, draw a line (AB) length c ($AB = BC = c$). Then draw BC line to AB the same length perpendicular. From point "A", we draw circles to the AC radius to cut the AB line at point "D". In the end, we draw a circle to center "B" and radius BD to cut AB line atpoint "E" and BC line at point "F". As indicated by the color in the image, the BE and BF segments are longitudinally (a) and the lines AE and CF are length (b).

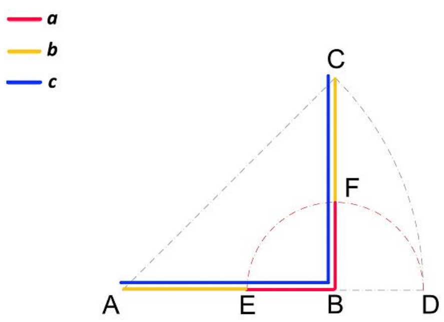

Figure 14: The first method is to plot lines with harmonic proportions.

- The second method:

  Again if we have three segments called a, b, c that is: $a < b < c$, There are harmonic proportions between them and only length c is specified. We use this geometric method to find the length of the lines a and b: As shown in

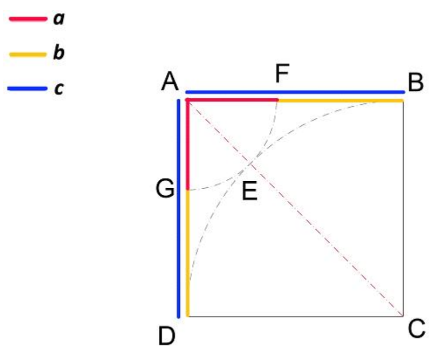

Figure 15: The second method is to plot lines with harmonic proportions.

  Figure 15, we draw the $ABCD$ square so that the length of the sides is equal to c. Then we draw a circle to center C and length c ($AB = AD = BC = CD = c$) to cut the square diameter at "E" point. In the end, draw a circle to center "A" and the AE radius to cut the side AB at point "F" and the side of AD in the "G". As indicated by the color in the image, the AF and AG segments are longitudinally (a) and the lines DG and BF are length (b).

As described in the first and second geometric drawing method, it is not necessary to use the formula for plotting the lines in which the relation between the harmonics is proportional and the lines can be easily drawn.

4. Conclusion

According to the material on musical and architectural commonality, it can be concluded that these two arts have many similarities in terms of structure and fundamental concepts, some of which are mentioned. The same adaptation allows the use of music to become an architectural form. Utilizing the abstract power of music in architecture can provide many architectural forms for the artist.

Table 3: Qualitative patterns of architecture and music.

Definition	Architecture	Music
Stagnation	A place to pause and stand like a square or park in the city	The silence
Movement	A place to move like a staircase, a ramp or a street	Playing music
Height	Short or high altitude in the building	Bass sound that depends on the frequency
Penumbra	Light penetration in the building	Up and down, subtly, or loud, or short, or a musical effect
Weight or Rhythm	Regular and frequent changes in color, materials, form of and dimensions of the building	Regular and frequent changes in the Silence or playing musical notes
Symmetry	Horizontal, vertical, or combining these two modes in form, geometry, materials and colors in the building	Horizontal or vertical symmetry, or combining these two modes in silence or playing musical notes
Line	A set of points that create a horizontal, vertical, or miles line in architecture	
Hierarchy	Crossing the spatial filters to get to a specific location in the architecture	Arrangement of items (The pitches and form of tonal music) to reach the peak of music
Proportions	Mathematical and geometric proportions in length, width and height in architecture	Mathematical and geometric proportions in the frequency of musical notes

Adaptive Table 3 is compiled as a summary of the common qualities of music and architecture. We have three dimensions in architecture. Length, width, and height; but the music formation basically has only one dimension, the frequencies are set to each other. If the ratio of frequencies to each other has three proportions of geometry, harmony and arithmetic, it creates beauty in a step. In order to achieve this goal, the proportions that are considered in the viewpoints of a building can be matched with the mathematical proportions of the musical steps to achieve the proper rhythm. In general, as music always relies on a particular step, a particular resistance, or a device, classical architecture is always based on a particular geometry. For example, we say that this piece is performed on the “Homayoon Mode”, that is, the specific musical intervals of the “Homayoon Mode”, and the corners of the device are shaped on the desired piece, and ignoring this matter, causes an unwanted result. It becomes obnoxious and ugly. In the classic and original architecture, the geometry of each design, role, form and plan should be followed, and the forms will be removed from their eye-catching and entertaining format [14]. One hopes that converting music to architecture using these parameters has the same effect on the audience and with the use of hierarchical patterns, rhythms, proportions and harmonies, and a small proportion of listening music to spectacular architecture and as Leon Battista Alberti (Italian, Genoa $1404 - 1472$ Rome) writes: "I believed in the truth of Pythagor as that nature undoubtedly works consistently. I conclude that the same numbers by which the hearing of the sounds on our ears are enjoyed coincidentally with our eyes and minds" [1].

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