Mathematics Interdisciplinary Research 4 (2019) 107 – 127

Golden Section in the Persian-Islamic Architecture Case Study: Hasht Behesht Palace, Isfahan, Iran

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Abstract

The subject of proportion used in architecture-and on a larger scale in any art work- is a debate having a special status in analytical studies on the history of art. "golden section" is known as one of the major topics of such debates. It has been given different names during the history of art, and it is generally defined under the ancient and Renaissance art in West.

The present paper studies the status of this specific kind of proportion in Islamic civilization. The authors also demonstrate their findings about the manifestation of this proportion in Hasht Behesht Palace, Isfahan. Moreover, they attempt to indicate the roots of familiarity with the usage of respective proportion in Muslim world by referring to some first-hand references of Islamic civilization in the fields of mathematics and geometry. The findings of this paper show that the application of golden section in Islamic civilization was independent of the developments of Western Renaissance and golden section had practical theorems in Islamic civilization since 10^{th} and 11^{th} centuries AD.

Keywords: Proportion, golden section, Nisbit Dhat Vasat Tarafein, Hasht Behesht Palace, Persian-Islamic architecture.

2010 Mathematics Subject Classification: $00A67$.

How to cite this article
R. Mojtahedzadeh and Z. Namavar, Golden section in the Persian-Islamic architecture, *Math. Interdisc. Res.* **4** (2019) 107 - 127.

1. Introduction

The present paper is a research on the subject of proportion, in general, and the golden section, in particular, in Islamic civilization and its applications in

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Academic Editor: Almantas Samalavicius
Received 27 May 2019, Accepted 20 July 2019
DOI: 10.22052/mir.2019.195272.1157

Islamic architecture. The research method of this study is interpretive-historical, conducted through library and field research. In addition to presenting a brief note about proportion in Islamic architecture, the attempt is also made to shed some light on the roots of the application of golden section in Islamic-Persian architecture through investigating the manifestation of this proportion in Hasht Behesht Palace of Isfahan. The close relation between Persia and West in Safavid era may lead to this hypothesis that golden section was introduced to the Safavid architecture through the concurrent developments in the Western art - Renaissance -; however, comprehensive studies indicate that this proportion was known and had been used by most Muslim prominent geometers and craftsmen long before Safavids. In fact, this proportion was known to Muslim scientists several centuries ahead of the book of "*De divina proportione*" (On the Divine Proportion) by Luca Pacioli in 1509 AD. At the very beginning, the Muslim geometers extracted such proportion from the Greek texts. Nevertheless, they played a significant role in transferring this proportion into the art and architecture of Islamic and Persian architecture through introducing practical theorems and the development of its practical applications.

2. Theoretical Aspects of Proportion in Islamic Architecture

There is no doubt that geometry is the essence of Islamic art, and particularly Islamic architecture. In the Islamic encyclopedias, architecture and its related professions along with mechanics have always been placed under the practical geometry. For instance, Abu Nasr Farabi in *ihsa' al-'ulum/An Encyclopedia of Sciences*, defines architecture as a geometrical branch of *Elm_ al_ Heial /Science of Tricks* which also includes surveying/land Surveying, making the war instruments, music instruments and etc [16]. Accordingly, the issue of proportion as one of the principal topics of geometry has a prominent status in architecture. Geometry have always had a bilateral role in Islamic civilization. On one hand, it was a means to inform measures and dimensions in the physical world, and on the other hand, it was tantamount to a ladder used to climb to the metaphysical world. Perhaps, the most evident reference in this field can be found in *Risa'el-i-Akhavan al-Safa/Epistles of the Brethren of Purity and Loyal Friends* by an anonymous author of the 10^{th} century AD/ 4^{th} century AH. In one section of this treatise, which is related to geometry, after enumerating some principles of Practical Geometry, which mainly had application in industry and architecture, the author(s) pointed out, "what we mentioned was just one part of physical geometry and it was an introduction to our main debate. Now the author(s) shall address one part of metaphysical geometry which has been one of the objectives for theologians and philosophers ... this type of geometry draws the learners' attention from materialistic affairs to spiritual ones" [15]. Therefore, it can be deduced that the issue of proportion- as one of the most significant branches of mathematics and geometry- was taken into consideration by Muslim scholars from both physical and metaphysical facets. Although there has been relatively little research specifically on the issue of proportion in Islamic architecture and civilization -in contrast to enormous researches conducted on the western ground of this matter-, an initial investigation by the authors indicated that the respective issue has never been an unknown matter to Muslim scholars, and a number of books and treatises on the topic identified.

One of the notable works in this field is *Rashikat al-Hend/Proportions among Indians* by Abu Rayhan Biruni, the renowned scientist of the $10-11^{th}$ centuries AD/$4-5^{th}$ century AH century, a summary of which presented in '*A Research on the Mathematic Works of Abu Rayhan Al-Biruni*' by A. Ghorbani. '*Rashikat al-Hind*' is a brief book in Arabic on the topic of proportion. The significant of this book is due to the fact that the author provided a new perspective of the subject of proportion by combining what he had founded in Indian mathematics in terms of ratio and proportion, with what had been reached to Muslims from Greek mathematics [7]. Thus, both Greek mathematical principals and Indian practical principals in solving proportion problems are simultaneously found in this book. At the beginning of the book, Biruni provided a definition of the concept of "proportion" which is an elaborated version of its definition in '*Principles of Euclid*' [7]. After enumerating various kinds of ratios between different shapes (e.g. the ratio between the diameter of a circle and its circumference or between square diameter and its side), Euclid's principles is alluded and it is mentioned that, " *'three'* is the minimum number of values among which proportion is feasible to be occurred", and that is the reason that 'three' is the minimum number of variables in any proportion [7]. How the aforementioned principle turns golden section to an exceptional proportion is illustrated in the following. In Biruni's book, the issue of proportion is mainly discussed from quantitative and mathematic perspective; however, as mentioned earlier, this issue has also a qualitative and spiritual facet, and it can be found in *Epistles of Akhavan al-Safa*, which is Biruni's Contemporary. There is a chapter in this epistles entitled "*The Virtue of Numerical, Geometrical and Musical Proportions*" discussing that all the universe, other than the almighty God, are of twofold essence and they are composed of different and contradictory elements. The authors stated that these contradictory elements fail to be gathered unless by compiling and composing, and the composition fails to create unity and amalgamation unless it is based on proportion [8]. Ibn Haytham, the renowned mathematician and physicist of the $10-11^{th}$ centuries AD, has a similar theory. In *Al-Manazer/The Optics Book*, after enumerating the influencing factors on beautify, such as light and color as the most effective ones, Ibn Al-Haytham introduced "proportion and coalition " as an additional factor and argued, "sometimes proportion, per se, can create beauty provided that its components, in turn, are not unpleasant, even though they may not be in perfect beauty status" [16]. Accordingly, it is evident that proportion, as a topic with various mathematical and aesthetical aspects, was a well-known subject for Muslim scientists, and subsequently it could have found its way in the professional communities, including architecture. There are various types of proportion used in art and architecture resulted from the variety of the shapes, applications, and patterns of these artifacts. One of the most prominent proportions widely recognized in the global history of art is "golden section". For the sake of better understanding of the importance of this proportion from conceptual perspective, it is required to present a brief definition of proportion and its different types, and it will be done by referencing *Sacred Geometry* by Robert Lawlor [14]. A proportion is formed from ratios and a ratio is a comparison of two sizes, quantities, qualities, or ideas, and is expressed by the formula $a : b$. However, a proportion is more complex, for it is a relationship of equivalency between two ratios, that is, the ratio of the first element to the second one, is similar to the ratio of the third element to the forth one (a is to b as c is to d or $a : b = c : d$). This relation was known in Greek thought as "***analogy***" and represents a level of intelligence more profound and precise than the direct response to a simple difference which is the ratio. Such analogy was frequently established among four different elements being analogous pairwise; thus, they were called "**discontinuous proportion**" of four terms. However, at times it may be possible to limit analogy to three terms ($a : b = b : c$), and subsequently lift the mind to a higher level because the reduction in the number of involved elements demands a more exacting determination. The Greeks called this mode a "***continuous proportion***" of three terms which previously referred by Biruni as "the minimum number of values among which proportion is feasible to be occurred [In the perspective of Greeks]". Due to its lower involved elements and the presence of linking element (b), continuous proportion of three was implied to be more closely to the concept of ***Unity***-which was a fundamental concept in the Greek mathematics and geometry- hence it was in a higher order compared to "discontinuous proportion" of four terms. In this manner, if a proportion is possible to be established between two terms, it possesses a higher level of exacting determination, focus and Unity. There is one, and only one, proportional division which is possible with two terms. Such a proportion -which was, indeed, an exception over the general rule presented by Biruni- is occurs when the smaller term to the larger term is equivalent to the larger term to the smaller plus the larger. Historically this unique geometric proportion of two terms was called "golden section" and -although it was known by cultures much older than the Greek- it is designated by the 21^{st} letter of the Greek alphabet, "ϕ", and numerically it equals to 1.618. In *Sacred Geometry*, Lawlor demonstrated that the golden section can be equal to 1, ($1/\phi + 1/\phi^2 = 1$), and from there he drew a number of absorbing conclusions fitting Christian philosophy and theology [14].

What may endow golden section with a unique importance in the history of art is the frequent and almost constant replication of this proportion in various components of the world around us; from plant's and animal's organs to the human artifacts and spatial-chronological intervals of natural elements. The abundance of such repetition has turned this specific proportion into one of the significant topics of investigation of proportion in the history of art. In the following, the attempt is made to examine the manifestation of this proportion in one of the distinguished monuments of Persian architecture, i.e. Hasht Behesht Palace of Isfahan, and subsequently explore the roots of the emergence of this proportion in the Persian-Islamic architecture. This subject is particularly important due to the fact that the role of golden section in Persian architecture has rarely discussed, and some resources have even introduced the Persian golden section in architecture as 1/73 (not 1/618) which is the ratio between the sides of the rectangle inscribed in a regular hexagon [18].

3. Case Study: Hasht Behesht Palace

Hasht Behesht Palace of Isfahan is one of the most celebrated royal palaces of the late Safavid era that has been continuously taken into the consideration by architects, art experts and travelers. Despite the relative lack of references of the second half of Safavid era−particularly the Shah Soleiman time and afterward−there are some descriptions in the literature of the time which indicate the importance of Hasht Behesht monument. The numerous lines of the poem in *Tazkarih-i-Nasabadi/Nasrabadi's* Biographers about the construction date of Hasht Behesht [10], and the long ode by Sa'eb Tabrizi describing the palace [11], positively demonstrate the eminence of the respective monument in its construction time. Furthermore, the references made by famed travelers in Shah Soleiman era, such as Engelbert Kaempfer, Jean Chardin and Nicolas Sanson, evidently shed some light on the prominence of this monument and its distinction from its contemporary counterparts. After presenting a relatively detailed description in terms of spatial components, the quasi−labyrinth quality of the space, and the unique features of Hasht Behesht Palace, Chardin stated, "Although this monument is not so durable and strong, it is more exhilarating than the most luxurious palaces in Europe" [4]. A relatively elongated chapter of Kaempfer's travelogue dedicated to describing the garden and palace of Hasht Behesht. Kaempfer, rather than Chardin, elaborated more on the spatial qualities of the palace and on the integration of water and light in its design. In describing the spatial features of Hasht Behesht, Kaempfer argued, "from either side you enter the building, you face the decorated and enchanting hall . . . water is streaming with a pleasant voice in a large pond and a ceiling-mounted wind-catcher permanently refreshes the air in the room. Space distribution between rooms is exceptionally skillful . . . the arrangement of arches and porches allows the wonderful landscape to be viewed from every direction . . . . In short, a man is required to have one hundred eyes to see the indescribable miscellaneous and splendor of the palace. In spite of all abovementioned remarks, the building is no more than thirty steps in length and width" [13]. The splendor of Hasht Behesht has not only been spectacular and attractive for its contemporaries, but also for all its eager visitors during its four hundred years history; Thus Hillenbrand, the renowned historian of Islamic architecture, believes that the building deserves the name of 'Hasht Behesht' (literally means eight paradises) due to the fantastic state of its spaces, and the wide view provided from inside the palace to the surrounding garden [9].

Hasht Behesht Palace is the type of *koushk* mansions with an old history in Persian architecture which traces back to the Achaemenian era and the remaining monuments in Pasargadae. *Koushk* is usually the main building of the garden mostly located at the middle, and at time at 1/3 of the length of the garden. Sometimes *koushk* opens to the garden from all four directions, and sometimes it has only one facade. The main facade of all types of the koushk is constantly facing the main axis of the garden [17]. In terms of location, Hasht Behesht is among middle−located *koushks* with four facades facing four cardinal points, and the main facade towards the North.

The external facade of Hasht Behesh Palace is simple and devoid of any specific embellishment or impressive decorative or formal elements; however, framing principles, solid and void arrangement, and specifically the lightness and transparency of the building is significantly eye-catching in the simple facade (Figure 1 and 2). The overall shape of the monument in outside is a cube with chamfered corners. Its plan is inscribed in 33.8×30.2 meter rectangle. Contrary to the exterior facade, the interior space of the palace is complex and asymmetric (Figure 3). Late Pirnia enumerated asymmetry −*Padjaftsazi*− as one of the common characteristic of most Persian palaces [17]; however, beyond the dominated asymmetry, Hasht Behesht demonstrates diversity in the special qualities and its components. Such diversity made a complexity in perceiving the interior space of the palace. This point was referred to in the reports delivered by Kaempfer and Chardin.

The internal complexity along with the external simplicity of the monument endow this building with ease yet difficult to imitate mode, which makes the analysis of the monument rather difficult. For instance, at the first glance, the palace is perceived as a square-like rectangle with long north-south axis. Further scrutiny reveals that, unexpectedly, the central hall, with rather similar proportion, has a long axis in east-west direction (Figure 4). A comparison between the plans of Hasht Behesh palace and the Taj Mahal, a mausoleum in India designed based on Persian and Timurid architectural traditions, demonstrates the complexity level of space design in Hasht Behesht Palace. Both abovementioned buildings follow the same *koushk* pattern, which is a cube with chambered corners divided into four zones by a cross at the middle of the plan. Contrary to the radial symmetry of Taj Mahal, with the central cross and the diameters of the inscribed square as its lines of symmetry, Hasht Behesht's plan demonstrates asymmetry, not only in the central cross and the size and form of the porches, but in the whole arrangement of the second floor (Figures 5 and 6). More geometrical analysis illustrated that the design is shaped by several rectangular space layers started from the central hall and expanded outwards. The expansion finally altered the long eat−west direction of the central hall to the long north−south axis of the the most external rectangle (Figure 7). The complexity of the design increases by looking into the repetition of a particular proportion used in the building. Each of the north, east and west porches of the palace have three columns dividing each porch into two equal parts at the sides and one larger part in the middle. Further examination indicated

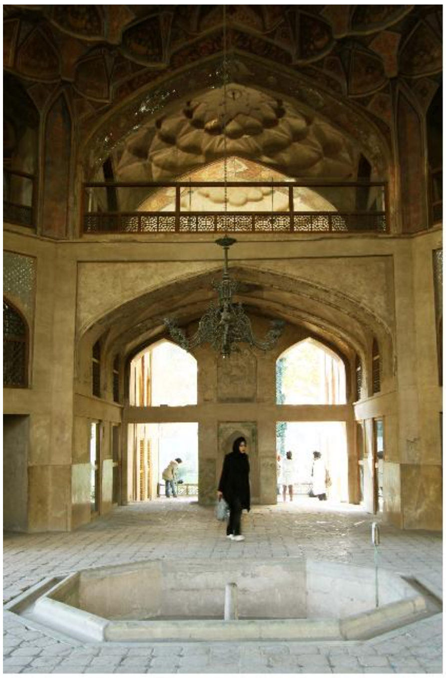

Figure 1: Central hall of Hasht Behesht Palace.

Figure 2: External view of the Hasht Behesht Palace.

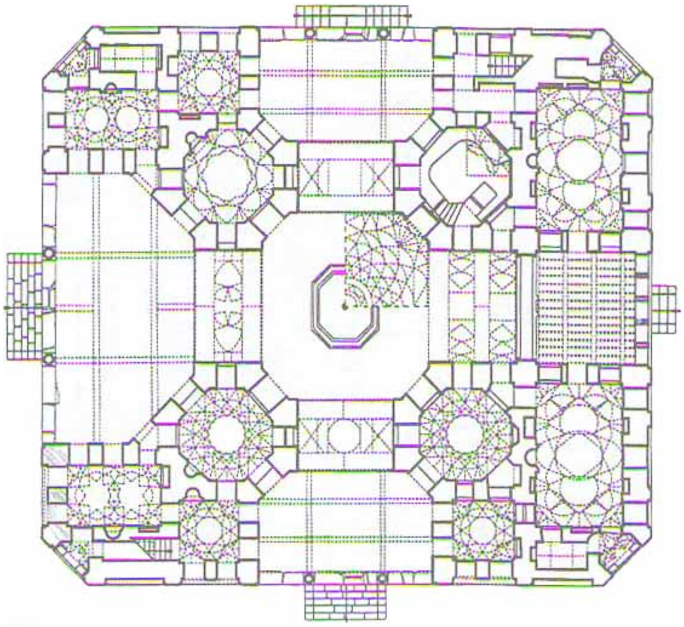

Figure 3: Plan of Hasht Behesht Palace [17].

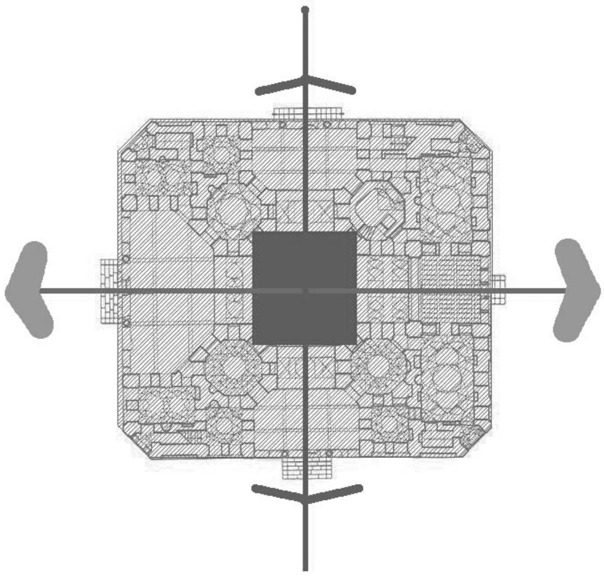

Figure 4: The external shape of the palace with long north-south axis, and the central hall with long east-west axis.

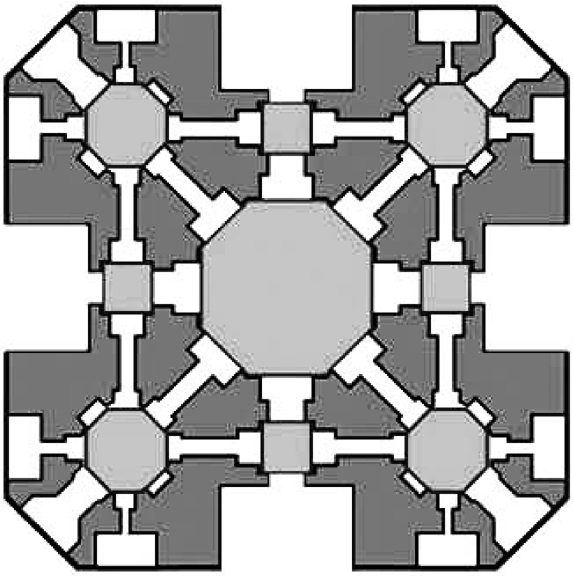

Figure 5: Plan of the Taj Mahal [17].

that the applied proportion between the small part and its large counterpart is golden section (Figure 8). Exact measurements revealed the presence of golden section in other parts of the palace, such as the distance between the beams of the roof, in determining the location of the pond in the central hall, structural divisions of the rooms and bridges of the second floor, between the length and the width of the south rooms (Figure 9). Likewise, the respective proportion exists between the size of the east and west porches with that of the south porch, which basically forms the negative cruciform space of the palace (Figure 10). In addition to plans, investigating the sections of the monument provided numerous samples of golden section applied in vertical directions. (Figure 11). The abundance of golden section, from the space organization to the ornaments, rejected any doubts with respect to the conscious application of this particular proportion in the design process of Hasht Behesht Palace. However, it raise questions about the reason for using such proportion in design and also about the extent to which the architects were aware of characteristic of this proportion and its mathematical principles. To answer these questions, the present study attempts to investigate the theoretical basis of golden section in Persian civilization, thus provides a better understanding of the concept and the usage of such proportion and its repetition in Hasht Behesht monument.

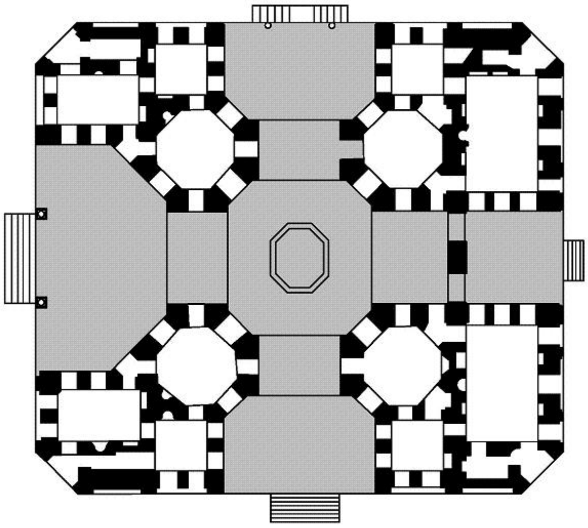

Figure 6: Plan of Hasht Behesht Palace.

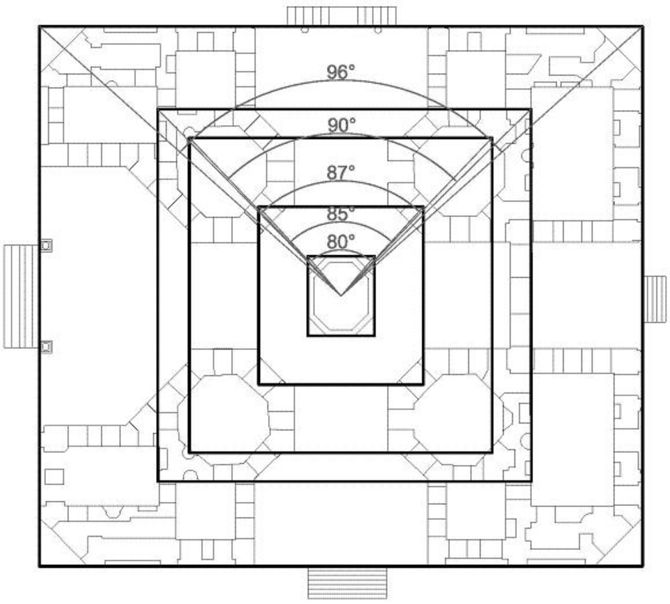

Figure 7: Rectangular spatial layering starts from the central hall and expanded outwards. The expansion finally altered the long north-south axis of the central hall to long east-west axis of the most external rectangle.

Figure 8: The north porch of Hasht Behesht Palace. The porch length is divided to three parts by the columns, and the proportion between its divisions is golden section.

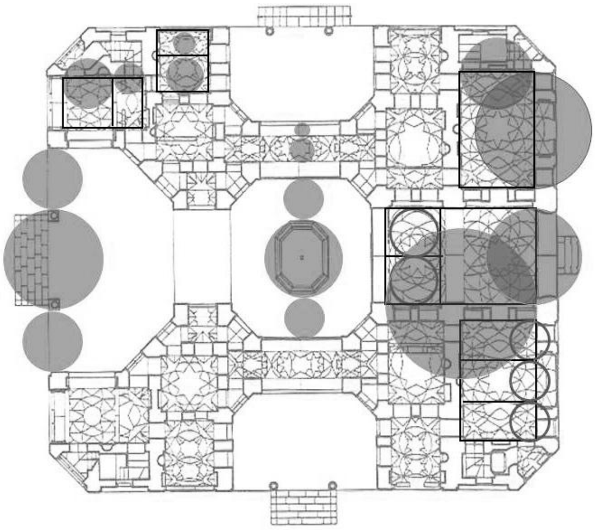

Figure 9: Manifestation of golden section in Hasht Behesht Palace.

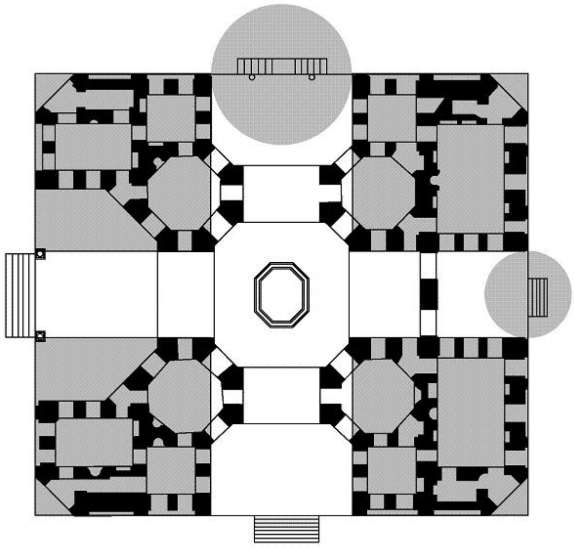

Figure 10: Golden section between the sizes of the porches.

4. Practical Geometry; a Link between Geometry and Architecture

Seeking the roots of the application of golden section in Islamic architecture necessitate a theoretical study of the first−hand resources related to Islamic architecture. However, the limited number −or relatively absence of such resources− is a fundamental problem in the process of studying Islamic architecture. Little existing resources in this ground, such as *Taq va Azaj/Vault and Arch* by Ghiyāth al-Dīn Jamshīd Al-Kāshī or *Geometry Būzhjānī Treatise*, have been mainly compiled with a practical approach, hence they rarely contain theoretic debates or discussions on design principles. However, the initial clues to understand whether the Muslim architects had been familiar with the golden section and its applications before European Renaissance era, was founded by the authors of this paper in aforementioned resources.

"*Practical Geometry*" by Abu'l-Wafa Al-Buzjani, the eminent scientist and mathematician of the 10^{th} century $AD/4^{th}$ century AH, is one of the few well-known books which, in some way, relates to the theoretical issues of Persian architecture. The full name of the book is "*Kitāb fī mā yahtāj ilayh al-sāni min al-amāl al-handasiyya/A Book on Those Geometric Constructions Which Are Necessary for a Craftsman*". As its name implies, the book did not specifically address architects; nevertheless, due to its simple language in explaining different geometrical issues, the book has been widely used in architecture. One of the presented problems in this book is how to draw a regular pentagon with given sides as AB by means of an open compass −a compass regulated at a fixed size. Buzjani has used golden section to solve the problem (Figure 12). Firstly, by applying geometrical methods, the golden section of the respective line was drew along it. Then, the respective *Mokhamas Triangle* of the side (an isosceles triangle with the respective side of the pentagon as its base and the pentagon vertex opposite to that side as its vertex) was obtained and the problem is solved [6]. In the same book, Buzjani presented a problem related to the pentagon star, i.e. a star created by drawing the pentagon's diameters. The divisions resulted from the intersection of diameters follow the golden section.

Figure 11: Several manifestations of golden section in the sections of Hasht Behesht Palace.

Abovementioned evidences led the authors of this paper to seriously investigate the likelihood of the familiarity of Muslim scientists, particularly mathematicians, with golden section. Two highly genuine evidences were found in the works of Biruni and Avicenna, the most prominent scientist and thinker of the Islamic Golden Age, lived in $10-11AD/4-5AH$ century. Those evidences demonstrate that respective proportion was profoundly known by Muslim scientists, and it was called "*Nisbat Zhat al-Vasat Tarafeyn/ The Middle and Two Sides Proportion*". In a relatively detailed introduction on the different types of mathematical and geometrical proportions in *al-Tafhim/Understanding Astrology*, Biruni has referred to *Nisbat Zhat al-Vasat Tarafeyn* and defined it as, "when a line is divided into two parts, in a way that the ratio of the small part to the large part equals the ratio of the large part to the whole" [1]. In the first section of *Kitab Al-Shifa/the book of healing* called 'The Principles of Geometry', Avicenna also dedicated two chapters to this proportion. One chapter is on the role of this proportion in regular polygons

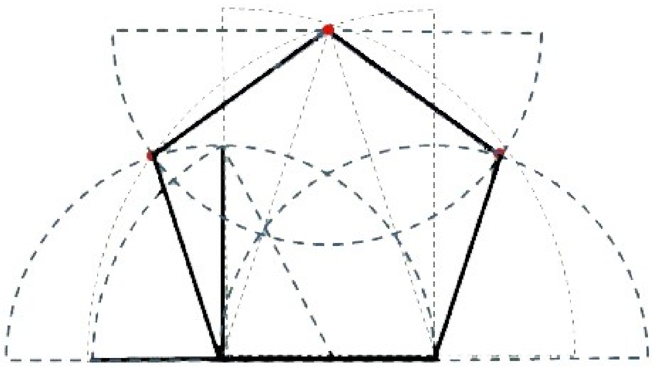

Figure 12: Buzjani's answer to the geometrical problem of 'how to draw a regular pentagon with given sides as AB by means of an open compass'. He used golden section to solve the problem.

and the other one on its role in the polyhedral. These two chapters, namely "*Nesbit Zhat al-Vasat Tarafeyn va al-Mozalaate al-Montazane*" and "*Nisbat Zhat al-Vasat Tarafeyn va al-Mojasamat al-Montazane*" are the 13^{th} and 14^{th} chapters of *Kitab Al-Shifa*. Apparently, the definition of golden section were such known at that time, that Avicenna did not seen necessity to provide any introduction or definition for this proportion in any of the book chapters. The thirteenth chapter is commenced with "In The Name of God" and then, without any introduction, followed by "the line was divided in point H according to *Nisbat Zhat al-Vasat Tarafeyn* [2]. This fact obviously indicates that in the 10^{th} century $AD/4^{th}$ century AH this proportion was perfectly known by Muslim scholars. However, the root of such familiarity is unknown. One of the possible roots might be the translation movement having been commenced two centuries ahead of Avicenna time in the world of Islam. *Principles of Euclid*, by the prominent ancient Greece scientist, is one of the oldest theoretical resources that demonstrated the popular golden section. In the second theorem of the second chapter of his book, Euclid presented a problem about dividing the given line AB into two sections, namely AG and BG, so that the area of a rectangle with AB and GB sides equals the area of a square with AG as its side [3]. In fact, this mathematics problem is another representation of the same popular golden section problem, mentioned earlier. This problem, along with several others from Greek mathematics, were known by Muslim mathematicians. *Principles of Euclid* had firstly been translated in the late 8^{th} century $AD/2nd$ century AH, followed by several other translations in the following decades, until the 9^{th} century $AD/3nd$ century AH witnessed a remarkable and rather perfect translation by Thābit ibn Qurra -prominent translator in Abbasid era [3]. All abovementioned evidences decisively assert that Muslim scientists were familiar with golden section since the $8^{th} - 9^{th}$ centuries AD and, likely, this is the time that they commenced to apply this proportion in different geometric works, particularly practical geometry. Example of the application of this proportion in the works of Avicenna was mentioned earlier in this paper. Likewise, examining the works of his contemporary scholars reveals similar findings. For instance, preliminary investigation in the works of Biruni, the authors found a theorem in "*Qanun al- Mas'udi /Mas'udi Canon*" −which is his most seminal books in astronomy− that could be a matter of great interest for architects and those who required the practical applications of geometric theorems. The theorem states, "If the radius of a circle is divided by the golden section, its larger section will happen to be a side of inscribed regular decagon" [7]. Such theorems could be directly used in architecture. Accordingly, it is not far-fetched to deduce that the emergence and spread of decuple divisions in monuments of medieval Islam, such as the dome of the *Gonbad-e* Qabus -one of the few remaining buildings of the early centuries of Islam in Iran, is attributed to the practical applications of these sorts of geometric theorems (Figure 13). Another evidence indicating the consistent interest of Muslims in Practical Geometry is the fact that Arabic version of the eighth chapter of *Pappus of Alexandria*, one of the first resources of Practical Geometry in Islam world, was re-edited by Abu Sa'eed Al-Sijzi, the great mathematician and astronomer of the10^{th} century $AD/4^{th}$ century AH [3]. A considerable part of the article was dedicated to the geometric drawings by means of an open compass −a compass regulated at a fixed size, which mentioned in Al-Buzjani's work earlier− and also included an elaborate section on the tools and devices to be used in various professions [3]. The content of the book indicates that the book had basically addressed craftsmen, rather than mathematicians. Furthermore, the fact that the re-edition of the book in the 10^{th} century $AD/4^{th}$ century AH coincided with the book of Nasr Farabi and the work of Buzjani, both on Practical Geometry, [3] indicates that not only the application of geometric theorems in architectural design was feasible since the 10^{th} centuries AD (4^{th} century AH, which is six hundred years before the Western Renaissance), but applied practical geometry allocated a special area of the knowledge of the time to itself. The continuity of such a trend− i.e. the *Practical Geometry*− is tractable until the 15^{th} century $AD/9^{th}$ century AH and the significant work *Miftah al-Hisab/Introduce on Mathematic* by Ghiyās-ud-dīn Jamshīd Kāshī. The forth article of the tenth chapter of this book was dedicated to compute the circumference and volume of the monuments and mansions. This chapter also included a comprehensive set of exercises related to various types of vaults, arcs, domes, and *Mogharnases.*

The issue of proportion and Practical Geometry has less taken into account in references of later periods. It can be due to the disappearance and negligence of such resources. It can also be due to their integration in the *fotovatnaméh* (a treatise or a book on the origins, beliefs, behaviors, and customs of professionals

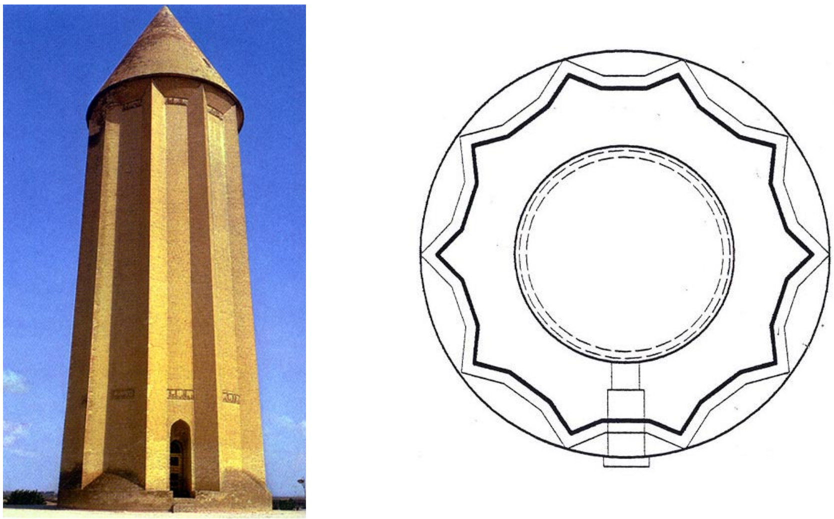

Figure 13: Gonbad-e Qabus, Iran (Photo: authors, Plan: Cultural Heritage Organization of Iran).

and the rules of their professions) or their integration in practical teachings of architects −which both of them are relatively unknown fields and require comprehensive studies.

All outlined evidences firmly asserts that golden section used in the design of Hasht Behesht Palace, were certainly known by the architect (s) of the monument and the repetition of such a proportion is by no means either accidental, unconscious, or imitative. Particularly because one specific composition of golden section, which was regularly used in the history of art, has been frequently observed in the palace. Such a special composition, called "golden cut" by Gyorgy Doczi, is classic structure includes a square with its golden rectangles on its two sides [5]. This structure is seen in a number of art and architectural works in different cultures and civilizations, from a Greek jug to Buddhist statues, and from Stonehenge and Roman monuments to temples of Mesopotamia and Japanese gardens. This composition can be observed in the central hall (*Houzkhane*), verandas, ornaments and structure of Hasht Behesht Palace (Figure 14).

Two years field and library research on Hasht Behesh palace along with geometrical analysis of the palace resulted in three theories regarding to its underlay geometry− which has been a system to control the whole design in Islamic art and architecture−. According to a number of criteria, such as historical evidences, the accuracy of superposing on the actual measurements, and adaptation with the palace's contemporary construction traditions, two theories were rejected. The third theory satisfactory met majority of the criteria. In addition, it was able to remove some ambiguity of the design process of the building [1] and gave more

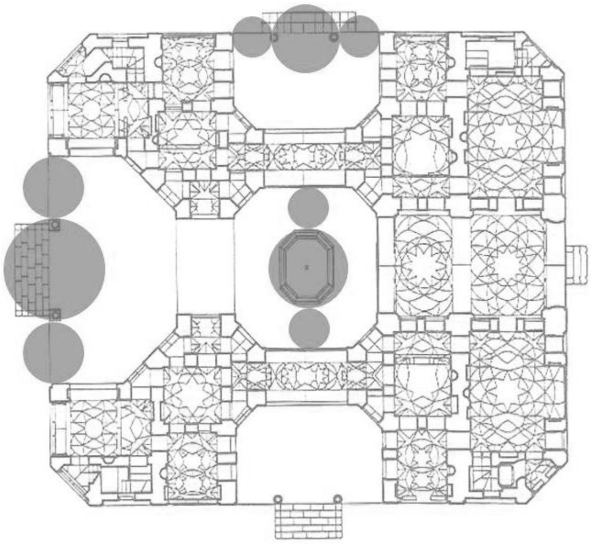

Figure 14: The manifestation of a classic structure of a square with its golden rectangles on its two sides in different parts of Hasht Behest Palace.

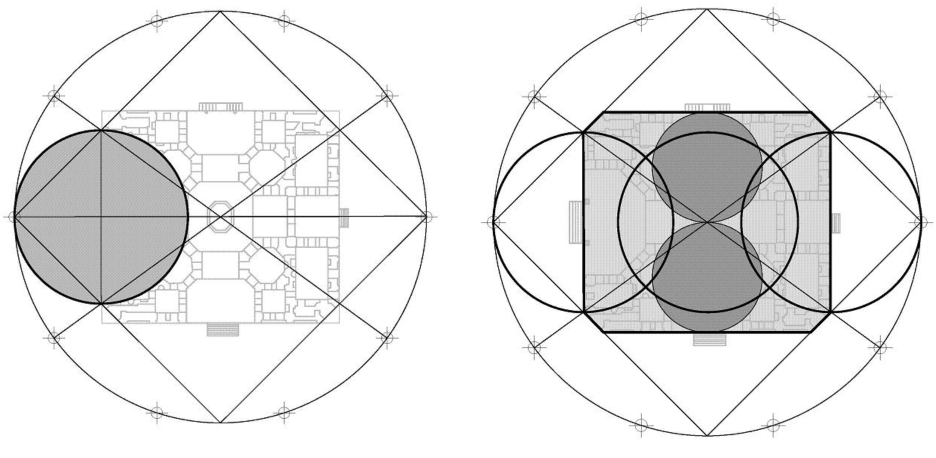

Figure 15: The theory which represents a circle divided into ten sectors as the base of the underlay geometry in Hasht Behesht Palace.

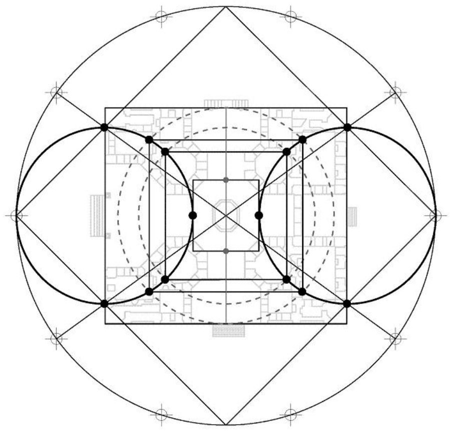

Figure 16: Superimpose of proposed underlay geometry and the rectangular space layers.

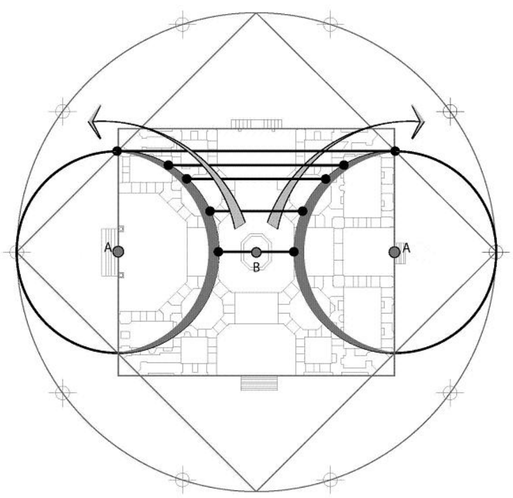

Figure 17: The two circles of the proposed underlay geometry are the locus of corners of the rectangles surrounded the space layering.

credibility to the theory. One of the significance capabilities of this theory was the answer it provided to find the geometry of expanding space layering in the building. It illustrated two circles as the locus of corners of the rectangles surrounding the space layers (Figures 16 and 17).

The theory presents a circle divided into ten sectors as the base of the underlay geometry (Figure 15). A circle divided into ten sections is a development of a circle divided into five sectors, hence acquiring the same characteristics. The circle divided to five equal sectors, as it discussed partly in Buzjani's works, forms a series of golden section in each level of its divisions, intersections of diameters, and geometrical developments. As mentioned earlier, the decagon was used widely in Islamic buildings at some point. Decagon, which is a development of pentagon, creates the 1.618 ratio in its divisions, similar to the 1.73 ration producing in hexagon-based geometry. Hexagon has always been a popular underlay geometry in Islamic architecture and art, thus the abundance of 1.73 ration in this art. In the same way, the decagon-based underlay geometry of Hasht Behesht might be the reason for the numerous manifestation of golden section in every aspects and level of design of the palace. However, the challenges -that is, whether such proportions was applied individually, or systematically and based on a specified patterns- is yet to be addressed. Comprehensive study on several case studies from different regions and different time spans is needed for providing an authentic answer.

5. Conclusion

Proportion, as one of the fundamental issues in designing and aesthetics of architecture, includes theoretical and metaphysical grounds in addition to physical and experimental aspects. The present study demonstrated that the familiarity with this facet of proportion in Islamic civilization goes back to the early After Hijrat centuries − known as the Islamic Golden Age, which Persian scholars had a considerable role in its formation−, and particularly the 10th century $AD/4^{th}$ century AH. This knowledge spread by the valuable works of renowned Persian scientists of the time including Biruni, Avicenna, and Ikhwan al-Safa to name a few, and found its way to architecture and other related professions through a number of treatises written on Practical Geometry.

The widely used golden section was also known by Muslims at that time. It was known as "Nisbat Zhat al-Vasat Tarafeyn" and a number of theorems in using such proportion in practice were provided. Accordingly, he application of this proportion and its related theorems in the monuments of Islamic architecture, particularly Persian−Islamic architecture in that era, was expected. The case study of this paper, Hasht Behesht Palace of Isfahan, is an example of the monuments which remarkably meeting such expectation. The abundant manifestation of this proportion in most elements and sections of the monument−in all three dimensions- pictures the palace as a monument surrounded by a halo of golden section. However, this matter may, in turn, raise some challenging questions, such as whether such proportions applied individually and merely according to the architect's decision, or systematically and based on some defined patterns? The above-mentioned question was tackled by the authors in another study, and some hypotheses drew; hence the hope for further developing the study and finding the opportunity to publish their findings in the future.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this article.

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