Mathematics Interdisciplinary Research **7** (2022) 1 − 19

Original Scientific Paper

On Quasi-Ordering Hypergroups, Ordered Hyperstructures and Their Applications in Genetics

Madeleine Al-Tahan and Bijan Davvaz⋆

Abstract

The study of hypercompositional structures (introduced by Marty) is now considered of a great value because of its applications in various sciences. In this paper, we focus on a special hypercompositional structure; quasi-ordering hypergroup. In this regard, we discuss some of the quasi-ordering hypergroup's properties and investigate some relations on it. Then, we present an application of quasi-ordering hypercompositional structures in genetics and define ordered hypercompositional structures for both sets: phenotypes and genotypes of F_2- offspring.

Keywords: quasi-ordering hypergroup, po-hypergroup, fundamental group, phenotype, genotype.

2020 Mathematics Subject Classification: 20N20.

How to cite this article
M. Al-Tahan and B. Davvaz, On quasi-ordering hypergroups, ordered hyperstructures and their applications in genetics, *Math. Interdisc. Res.* **7** (2022) 1 − 19.

1. Introduction

Since its introduction around 80 years ago, hypercompositional structures theory has become an important area of research. In which researchers, and besides the theoretical part, have been discussed its applications in various areas (see [10]).

⋆Corresponding author (E-mail: davvaz@yazd.ac.ir)
Academic Editor: Ali Reza Ashrafi
Received 6 August 2021, Accepted 5 February 2022
DOI: 10.22052/mir.2022.242993.1299

Several books and articles were written on the concepts and applications of hypercompositional theory (see [13, 21, 25, 26, 28]). Hypercompositional structures are a natural extension of the ordinary algebraic structures. In an ordinary algebraic structure, elements are composed to produce a single element, whereas in hypercompositional structures, elements are composed to produce a non-void set. Marty [22], who introduced this important concept in 1934, is considered as the pioneer of hypercompositional structures theory. Since then, this theory was developed by many researchers and thus new definitions were introduced like cyclic hyperstructures [30] and studied by many researchers (see [1, 2, 3, 14, 20]) and ordered hypercompositional structures. The main link connecting the class of hypercompositional structures with that of classical algebraic structures is fundamental relations.

Bakhshi and Borzooei [6] introduced the concept of ordered polygroups. Later, Davvaz and Heidari generalized the concept of ordered semigroups to ordered semihypergroups in [16]. Chvalina in [7] and Hort in [17] used ordered hypercompositional structures for the construction of hypergroups. Since then, ordered hypercompositional structures have been studied by different researchers (see [8, 18, 27]). As an example, Omidi and Davvaz introduced in [27] the concept of ordered (semi)hyperrings and used the notion of pseudo-order on an ordered (semi)hyperring to obtain (semi)rings.

The laws of inheritance were put by Mendel in 1866 who first traced patterns of certain pea plants' traits and proved that some statistical rules were valid [24]. A connection between hypercompositional structures and inheritance was established by Davvaz et al. in [11] where they provided some inheritance examples of algebraic hypercompositional structures. The authors in [4] generalized the connection related to phenotypes and found another connection related to genotypes in [5].

The manuscripts aims at studying quasi-ordering hypercompositional structures and finding a connection between it and biological inheritance. The remaining part of the manuscript is as follows: Section 2 covers the main concepts that are needed throughout the paper. Sections 3 discusses quasi-ordering hypergroups and studies its properties. Section 4 deals with equivalence relations of quasi-ordering hypergroups. Section 5 presents an application of hypercompositional structures in genetics found by the authors in [4, 5], proves that one of the hyperstructures defined before is a po-ordering hypergroup and the other is a po-H_v-semigroup.

Throughout the manuscript, P is used for parents, F_1 for first generation, F_2 for second generation, and the genotypes ab, ba are considered equal.

2. Basic Notions and Concepts

This section presents the needed definitions related to both: hypercompositional structures (see [1, 12, 20]) and biological inheritance (see [24]).

2.1 Hypercompositional Structures

Definition 2.1. [12] Let $H \neq \emptyset$ be any set and $\mathcal{P}^*(H) = \{X \neq \emptyset : X \subseteq H\}$. Then a *binary hyperoperation* on H is a mapping $\divideontimes : H \times H \to \mathcal{P}^*(H)$ and $(H, \divideontimes)$ is called a *hypergroupoid*.

In Definition 2.1, if K and L are non-void subsets of H and $\gamma \in H$, we have:

$$K \divideontimes L = \bigcup_{\substack{k \in K \\ l \in L}} k \divideontimes l,\ \gamma \divideontimes L = \{\gamma\} \divideontimes L \text{ and } K \divideontimes \gamma = K \divideontimes \{\gamma\}.$$

As an extension of hypercompositional structures, T. Vougiouklis [29] introduced weak hypercompositional structures (H_v-structures). Where weak axioms in the latter replace some axioms of classical hypercompositional structures.

An H_v*-semigroup* is a hypergroupoid $(H, \divideontimes)$ satisfying $(\alpha \divideontimes (\beta \divideontimes \gamma)) \cap ((\alpha \divideontimes \beta) \divideontimes \gamma) \neq \emptyset$ for all $\alpha, \beta, \gamma \in H$. An H_v*-subsemigroup* R of an H_v-semigroup H is a non-void subset of H satisfying $R \divideontimes R \subseteq R$. An element $\varrho \in H$ is called an *identity* of $(H, \divideontimes)$ if $\varrho \in \alpha \divideontimes \varrho \cap \varrho \divideontimes \alpha$, for all $\alpha \in H$ and it is said to be *scalar identity* of $(H, \divideontimes)$ if $\alpha \divideontimes \varrho = \varrho \divideontimes \alpha = \{\alpha\}$, for all $\alpha \in H$. An *idempotent* is an element $\alpha \in H$ satisfying $\alpha^2 = \alpha \divideontimes \alpha = \alpha$. A hypergroupoid $(H, \divideontimes)$ is *commutative* if $\alpha \divideontimes \beta = \alpha \divideontimes \beta$, for all $\alpha, \beta \in H$. A *semihypergroup* is a hypergroupoid $(H, \divideontimes)$ satisfying the associative law and a *quasihypergroup* is a hypergroupoid satisfying the reproduction axiom, i.i, $\alpha \in H$, $\alpha \divideontimes H = H = H \divideontimes \alpha$. A hypergroupoid that is both: a semihypergroup and a quasihypergroup is called a *hypergroup*. The *total hypergroup* is an example of hypergroups where $\alpha \divideontimes \beta = H$ for all $\alpha, \beta \in H$. A *regular hypergroupoid* is a hypergroupoid with at least one identity and each element admits one inverse (or more). A non-void subset R of a hypergroup $(H, \divideontimes)$ satisfying the reproduction axiom is called *subhypergroup*. And if $\alpha \divideontimes R = R \divideontimes \alpha$ for all $\alpha \in H$ then R is a *normal subhypergroup* of H. A non-void subset I of H is called a *hyperideal* of H if $(I \divideontimes H) \cup (H \divideontimes I) \subseteq I$. If no proper hyperideals exist then the hypergroup is *simple*.

Let $(H, \divideontimes)$ and $(K, \star)$ be hypergroups (H_v-semigroups or H_v-groups). Then H and K are *isomorphic hypergroups* if there exists a bijective function $\phi : H \to K$ with $\phi(\alpha \divideontimes \beta) = \phi(\alpha) \star \phi(\beta)$ for all $\alpha, \beta \in H$.

Definition 2.2. [12] A non-void subset L of a hypergroup $(H, \star)$ is linear if for all $\alpha, \beta \in L$, $\alpha \star \beta \subseteq L$ and $\alpha/\beta = \{\gamma \in H \mid \alpha \in \gamma \star \beta\} \subseteq L$.

Definition 2.3. [23] Let $(H, \divideontimes)$ be a hypergroupoid with at least one identity ϱ. An element $\alpha \in H$ is called ϱ*-attractive* if $\varrho \in (\varrho \divideontimes \alpha) \cap (\alpha \divideontimes \varrho)$.

Definition 2.4. [23] Let $(H, \divideontimes)$ be a hypergroupoid with an identity ϱ. Then ϱ is called *strong identity* if $\varrho \in \varrho \divideontimes \alpha = \alpha \divideontimes \varrho \subseteq \{\varrho, \alpha\}$.

A hypergroupoid H is *cyclic* if for some $\alpha \in H$,

$$H = \alpha \cup \alpha^2 \cup \cdots \cup \alpha^n \cup \cdots .$$

If there exists a minimum positive integer m satisfying the following property

$$H = \alpha \cup \alpha^2 \cup \cdots \cup \alpha^i,$$

then H is a cyclic hypergroupoid with finite period and α is a generator of H with period m.

If there exists a minimum positive integer m satisfying $H = \alpha^i$ then H is a *single-power cyclic hypergroupoid* and α is a generator of H with period m (For more details, see [30]).

2.2 Regular Relations and Complete Parts

Definition 2.5. [9, 12] Let R be an equivalence relation on the semihypergroup $(H, \circledast)$. For $X, Y \neq \emptyset \subseteq H$,

1. by $X\overline{R}Y$, it is meant that for every $x \in X$ there exists $y \in Y$ satisfying xRy and for every $y' \in Y$ there exists $x' \in X$ satisfying $x'Ry'$;
2. by $X\overline{\overline{R}}Y$, it is meant that xRy for every $x \in X$ and $y \in Y$.

Moreover, R is:

1. a regular on the right (on the left) if for all $\alpha \in H$, xRy implies that that $(x \circledast \alpha)\overline{R}(y \circledast \alpha)$ $((\alpha \circledast x)\overline{R}(\alpha \circledast y))$;
2. a strongly regular on the right (on the left) if for all $\alpha \in H$, xRy implies that $(x \circledast \alpha)\overline{\overline{R}}(y \circledast \alpha)$ $((\alpha \circledast x)\overline{\overline{R}}(\alpha \circledast y))$;
3. a regular (strongly regular) if it is regular (strongly regular) on both: the left and the right.

Theorem 2.6. [9, 12] *Let $(H, \circledast)$ be a hypergroup, R an equivalence relation on H, and H/R be the set of all equivalence classes. Then R is strongly regular if and only if $(H/R, \otimes)$ is a group.*

Definition 2.7. Let $n > 1$ be an integer, $(H, \circledast)$ be a semihypergroup, and β_n be the relation on a H given as follows:

$$x\beta_n y \text{ if there exist } x_1, \ldots, x_n \text{ in } H \text{ such that } x, y \in \prod_{i=1}^{n} x_i.$$

Here, $\beta_1 = \{(a, a) \mid a \in H\}$ and $\beta = \bigcup\limits_{n \geq 1} \beta_n$.

The above symmetric and reflexive relation was introduced by Koskas [19] and studied by many scholars such asFreni, Davvaz, Corsini, Leoreanu-Fotea, and Vougiouklis [9, 12, 15, 29].

The *fundamental equivalence relation* β^* on H is the transitive closure of β and it is the smallest strongly regular relation on H. Moreover, H/β^* is called the *fundamental group.* For a hypergroup H, $\beta = \beta^\star$ [15].

Definition 2.8. [19] Let $(H, \divideontimes)$ be an H_v-group and $C \neq \emptyset \subseteq H$. Then C is a complete part of H if for every positive integer m and for all $P \in H_H(m) = \{\prod_{i=1}^{m} x_i : x_i \in H\}$, we have $C \cap P \neq \emptyset \Longrightarrow P \subseteq C$.

2.3 Ordered Hypercompositional Structures

Definition 2.9. [7] A hypergroup $(H, \divideontimes)$ is called a *quasi-ordering hypergroup* if for all $x, y \in H$, the following conditions hold.

1. $x \divideontimes y = x^2 \cup y^2$,
2. $x \in x^2 = x^3$.

The quasi-ordering hypergroup $(H, \divideontimes)$ is an *ordering hypergroup* if $x^2 = y^2 \Longrightarrow x = y$ holds.

Definition 2.10. [16] Let $(H, \divideontimes)$ be a semihypergroup and $\leq$ a partially preordered (order) relation on H satisfying the monotone condition, i.e.,

$$x \leq y \Longrightarrow \gamma \divideontimes x \leq \gamma \divideontimes y \text{ and } x \divideontimes \gamma \leq y \divideontimes \gamma \text{ for all } x, y, \gamma \in H.$$

Then $(H, \divideontimes, \leq)$ is a partially preordered (ordered) semihypergroup.

The term po-semihypergroup is used for partially ordered semihypergroup and the term po-hypergroup is used for partially ordered hypergroup.

Theorem 2.11. [16] *A quasi-ordering hypergroup $(H, \divideontimes, \leq)$ is a partially preordered hypergroup and an ordering hypergroup $(H, \divideontimes, \leq)$ is a po-hypergroup. Here, $\alpha \leq \beta \Longleftrightarrow \alpha \in \beta^2$ for all $\alpha, \beta \in H$.*

Definition 2.12. [16] Let $(H, \divideontimes, \leq)$ be a po-semihypergroup and $B \subseteq H$. Then

1. $(B] = \{x \in H : x \leq b, \text{ for some } b \in B\}$;
2. $< B >= (B] \cup (H \divideontimes B] \cup (B \divideontimes H] \cup (H \divideontimes B \divideontimes H]$.

If $(H, \divideontimes)$ is commutative then $< B >= (B] \cup (H \divideontimes B]$.

2.4 Biological Inheritance

The genetic information passing for traits from parents to their offsprings is called *inheritance*. Autosomal inheritance is not affected by the sex of the parents. It is a pattern of inheritance in which the transmission of traits depends on the presence or absence of certain alleles. Inherited traits are controlled by genes and a *genotype* is the complete set of genes within an organism's genome. A *phenotype* is the complete set of observable traits of the structure and behavior of an organism.

In 1865, teh concept of inheritance was introduced explicitly by Gregor Mendel [24]. He found that paired pea traits were either dominant or recessive. The inheritance factors are alleles that are different variants of the same gene. A homozygote has two identical copies of the same allele whereas a heterozygote has two different alleles.

3. Properties of Quasi-Ordering Hypergroups

This section discusses a commutative hypergroup defined by Chvalina in [7] and proves some results regarding its properties.

Example 3.1. Let $H \neq \emptyset$ be any set and define $(H, \circledast_1)$ and $(H, \circledast_3)$ as the total hypergroup and Biset-hypergroup (B- hypergroup, i.e. $\alpha \circledast_1 \beta = \{\alpha, \beta\}$ for all $\alpha, \beta \in H$.) respectively. It is clear that $(H, \circledast_1)$ is a quasi-ordering hypergroup that is not an ordering hypergroup. Whereas, $(H, \circledast_3)$ is an ordering hypergroup.

Remark 1. Let $(H, \circledast)$ be any quasi-ordering hypergroup. Then for all $a, b \in H$,

$$a \circledast_3 b \subseteq a \circledast b \subseteq a \circledast_1 b.$$

Example 3.2. Let $H = \{h_1, h_2\}$ and define $(H, \circledast_2)$ as follows:

$\circledast_2$	h_1	h_2
h_1	H	H
h_2	H	h_2

It is clear that $(H, \circledast_2)$ is an ordering hypergroup.

Remark 2. Since every ordering hypergroup is a quasi-ordering hypergroup, it follows that properties of quasi-ordering hypergroups are applied to ordering hypergroups.

Proposition 3.3. *A quasi-ordering hypergroup is regular.*

Proof. Let $(H, \circledast)$ be a quasi-ordering hypergroup and $\alpha, \beta \in H$. Definition 2.9 asserts that $\alpha \in \alpha \circledast \beta = \alpha^2 \cup \beta^2$. Thus, each element in H is an identity. The latter implies that the set of all inverses $I(\alpha)$ of $\alpha \in H$ is equal to H. $\square$

Proposition 3.4. *Let $(H, \circledast)$ be a quasi-ordering hypergroup then for all $x, y \in H$, y is x-attractive.*

Proof. The proof is straightforward. $\square$

Proposition 3.5. *Let $(H, \circledast)$ be a quasi-ordering hypergroup having a strong identity e. Then $x^2 = x$ or $x^2 = \{e, x\}$ and $x \circledast y = \{x, y\}$ or $\{x, y, e\}$.*

Proof. Having that $\{e, \alpha\} \supseteq \alpha \circledast e = \alpha^2 \cup e^2$ and $e^2 = e$ implies that $\alpha^2 = \alpha$ or $\alpha^2 = \{e, \alpha\}$. Consequently, $\alpha \circledast \beta = \{\alpha, \beta\}$ or $\{\alpha, \beta, e\}$. $\square$

Proposition 3.6. *Let $(H, \circledast)$ be a non-trivial quasi-ordering hypergroup. Then $(H, \circledast)$ doesn't admit a scalar identity.*

Proof. Let $e \in H$ be a scalar identity and $x \neq e$ be an element in H. Then $e^2 = e$ and $x \circledast e = x$. The latter implies that $x \circledast e \neq x^2 \cup e^2$. $\square$

Example 3.7. There are three quasi-ordering hypergroups of order two (up to isomorphism) given by $(H, \circledast_1)$, $(H, \circledast_2)$ and $(H, \circledast_3)$ (defined in Examples 3.1 and 3.2).
It is clear that $(H, \circledast_1)$ has no strong identity, $(H, \circledast_2)$ has a unique strong identity h_2 and $(H, \circledast_3)$ has two strong identities h_1 and h_2.

Remark 3. Strong identities in quasi- ordering hypergroup need not to be unique ($(H, \circledast_3)$ in Example 3.7).

Proposition 3.8. *A quasi-ordering hypergroup has no proper linear subsets.*

Proof. Let L be a linear subset of the quasi-ordering hypergroup $(H, \circledast)$ and $a \in L$. Having $a/a \subseteq L$ implies that $a/a = \{x \in H : a \in x \circledast a\}$ and hence, $a/a = H \subseteq L$. □

Proposition 3.9. *Let $(H, \circledast)$ be a quasi-ordering hypergroup and $(S, \circledast)$ a subhypergroup of $(H, \circledast)$. Then $(S, \circledast)$ is a quasi-ordering hypergroup.*

Proof. The proof is straightforward. □

Proposition 3.10. *Let $(H, \circledast)$ be a quasi-ordering hypergroup and $S \subseteq H$. If $s^2 \subseteq S$ for all $s \in S$ then $(S, \circledast)$ is a subhypergroup of $(H, \circledast)$.*

Proof. We need to show that $(S, \circledast)$ satisfies the reproduction axiom. We have that $x \in x \circledast S = \{x^2 \cup y^2 : y \in S\} \subseteq S$ for all $x \in S$. Therefore, $x \circledast S = S$. □

Proposition 3.11. *A quasi-ordering hypergroup has no proper normal subhypergroups.*

Proof. Let N be a normal subhypergroup of the quasi-ordering hypergroup $(H, \circledast)$ and $x \in H$. Since $x \in x \circledast N = \{x^2 \cup n^2 : n \in N\}$, it follows that $N = H$. □

Proposition 3.12. *A quasi-ordering hypergroup is simple.*

Proof. One can easily see that a quasi-ordering hypergroup has no proper hyperideals and hence, it is simple. □

Proposition 3.13. *A cyclic quasi-ordering hypergroup $(H, \circledast)$ is a single-power cyclic hypergroup of period two. Moreover, if h is a generator of it then $h \circledast x = H$ for all $x \in H$.*

Proof. Since $(H, \circledast)$ is cyclic and h is a generator of H, it follows that $H = h \cup h^2 \cup \ldots$. Definition 2.9 implies that $h \in h^2 = h^3$. It is easy to see that $h^i = h^2$ for all $i \geq 2$. Thus, $H = h^2$. Moreover, $h \circledast x = h^2 \cup x^2 = H$ for all $x \in H$. □

Corollary 3.14. *A cyclic ordering hypergroup then $(H, \circledast)$ has only one generator.*

Proof. Let h, x be generators of $(H, \circledast)$ then Proposition 3.13 asserts that $H = h^2 = x^2$. Thus, $h = x$ by Definition 2.9. $\square$

Example 3.15. Let H be any set with a least three elements x, y, z and define $(H, \circledast)$ as follows:

$$x^2 = x \circledast y = H \text{ and } y \circledast z = \{y, z\} \text{ for } y, z \neq x.$$

Then $(H, \circledast)$ is an ordering hypergroup.

Proposition 3.16. *Let $(H, \circledast)$ be a quasi-ordering hypergroup, $(K, \star)$ be any hypergroup, and $f : H \longrightarrow K$ be an isomorphism. Then $(K, \star)$ is a quasi-ordering hypergroup. Moreover, if $(H, \circledast)$ is an ordering hypergroup then $(K, \star)$ is an ordering hypergroup.*

Proof. For all $(y_1, y_2 \in K$ there exists $(x_1, x_2 \in H$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. We have:

1. $y_1 \star y_2 = f(x_1) \star f(x_2) = f(x_1 \circledast x_2) = f(x_1 \circledast x_1 \cup x_2 \circledast x_2) = f(x_1 \circledast x_1) \cup f(x_2 \circledast x_2)$. Having f a good homomorphism implies that
$$f(x_1 \circledast x_1) \cup f(x_2 \circledast x_2) = f(x_1) \star f(x_1) \cup f(x_2) \star f(x_2) = y_1 \star y_1 \cup y_2 \star y_2.$$
2. $y_1 \star y_1 = f(x_1) \star f(x_1) = f(x_1 \circledast x_1)$. And having $(H, \circledast)$ a quasi-ordering hypergroup implies that $x_1 \in x_1^2 = x_1^3$. Thus, $y_1 = f(x_1) \in f(x_1^2) = f(x_1^3)$. The latter and having f a good homomorphism imply that $y_1 \in y_1^2 = y_1^3$.

Thus, $(K, \star)$ is a quasi-ordering hypergroup.

Let $(H, \circledast)$ be an ordering hypergroup and $y_1^2 = y_2^2$. Then $f(x_1^2) = (f(x_1))^2 = (f(x_2))^2 = f(x_2^2)$. Having f an injective function implies that $x_1^2 = x_2^2$. Since $(H, \circledast)$ is an ordering hypergroup, it follows that $x_1 = x_2$ and thus $y_1 = y_2$. $\square$

4. Fundamental Groups and Regular Relations of Quasi-Ordering Hypergroups

This section studies equivalence relations on quasi-ordering hypergroups, finds its fundamental group, and determines its complete parts.

Proposition 4.1. *The fundamental group of a quasi-ordering hypergroup is the trivial group.*

Proof. Let $(H, \circledast)$ be a quasi-ordering hypergroup and $x, y \in H$. Then $\{x, y\} \subseteq x \circledast y$. We get that $x\beta_2 y$ and hence, $x\beta y$. Having $(H, \circledast)$ a hypergroup implies that β and $\beta^\star$ coincide. Therefore, $H/\beta^\star$ is the trivial group. $\square$

Proposition 4.2. *A quasi-ordering hypergroup has no proper complete parts.*

Proof. Let C be a complete part of the quasi-ordering hypergroup $(H, \circledast)$ and $a \in C$. For all $b \in H$, we have $a \in C \cap (a \circledast b) \neq \emptyset$. Since C is a complete part of H, it follows that $b \in a \circledast b \subseteq C$. $\square$

Proposition 4.3. *Let $(H, \circledast)$ be a quasi-ordering hypergroup and R an equivalence relation on it. Then R is a strongly regular relation on H if and only if H/R is the trivial group.*

Proof. The proof is straightforward. $\square$

Proposition 4.4. *Let $(H, \circledast)$ be a quasi-ordering hypergroup and R be the relation on H given as follows:*

$$aRb \Longleftrightarrow a^2 = b^2 \text{ for all } (a, b) \in H^2.$$

Then R is a regular relation on H. Moreover, R is strongly regular relation on H if and only if $(H, \circledast)$ is the total hypergroup.

Proof. It is clear that R is an equivalence relation on H. Let $(a, b, x, \alpha) \in H^4$, aRb and $\alpha \in a \circledast x = a^2 \cup x^2$. The latter implies that $\alpha \in a^2 = b^2$ or $\alpha \in x^2$. Thus, $\alpha \in b^2 \cup x^2 = b \circledast x$. Therefore, $a \circledast x \overline{R} b \circledast x$.

It is clear that if $(H, \circledast)$ is a total hypergroup then R is strongly regular relation on H. Let $a \in H$. For a strongly regular relation R on H, we have aRa implies that $(a \circledast z)\overline{\overline{R}}(a \circledast z)$ for all $z \in H$. Since $\{z, a\} \subseteq a \circledast z$, it follows that aRz. Thus $a^2 = z^2$ for all $a, z \in H$. Definition 2.9 asserts that $a^2 = a \circledast z$ for all $z \in H$. The reproduction axiom implies that $a \circledast z = H$. $\square$

5. Applications of Quasi-Ordering and Ordered Hyperstructures in Genetics

This section considers the hyperstructures defined by the authors in [4] and [5] and defines ordered hyperstructures on it.

Throughout this section, the hyperoperations"$\otimes$" and "$\times$": represent the mating.

5.5 Hyperstructures and Biological Ineritance

We present the results of the authors in [4] and [5].

5.5.1 Phenotypes of F_2-offspring

The authors in [4] presented the following example of non-Mendelian inheritance.

Example 5.1. [4] *Epistasis of dominant gene in dogs' coat color.* Epistasis occurs when genes at two different loci interact to affect the expression of a single trait. We have: A is dominant over a, B is dominant over b, $AxBy$ and $Axbb$ have phenotype white, and $aaBy$ has phenotype black, and $aabb$ has phenotype brown. Here $x \in \{A, a\}$ and $y \in \{b, b\}$. This experiment's results are presented as follows.

$$P : AABB \otimes aabb$$
$$F_1 : AaBb$$
and
$$F_1 \otimes F_1 : AaBb \otimes AaBb$$
$$F_2 : \text{White, Black, Brown.}$$

White is denoted by A_1, Black by A_2, and Brown by A_3.

By setting $H = \{A_1, A_2, A_3\}$, $(H, \otimes)$ is presented in the following table.

$\otimes$	A_1	A_2	A_3
A_1	H	H	H
A_2	H	$\{A_2, A_3\}$	$\{A_2, A_3\}$
A_3	H	$\{A_2, A_3\}$	$\{A_3\}$

.

The authors in [4] proved that $(H, \otimes)$ is a hypergroup. Moreover, they considered the hypergroup of phenotypes in F_2-offspring under simple inheritance and studied it in details. More specifically, the $n-$hybrid cross ($n \geq 1$) that differs in n traits; a homozygous dominant parent $(A_1A_1A_2A_2 \dots A_nA_n) \otimes$ a homozygous recessive parent $(a_1a_1a_2a_2 \dots a_na_n)$. This experiment's results is summarized as follows.

$$P : A_1A_1A_2A_2 \dots A_{n-1}A_{n-1}A_nA_n \otimes a_1a_1a_2a_2 \dots a_na_n$$
$$F_1 : A_1a_1A_2a_2 \dots A_na_n$$
and
$$F_1 \otimes F_1 : A_1a_1A_2a_2 \dots A_na_n \otimes A_1a_1A_2a_2 \dots A_na_n$$
$$F_2 : \widehat{A_1}(\text{of genotype} A_1x_1A_2x_2 \dots A_nx_n), \dots,$$
$$\widehat{A_r}(\text{of genotype } A_1a_1A_2a_2 \dots A_{n-1}a_{n-1}A_na_n), \dots,$$
$$\widehat{A_s}(\text{of genotype} a_1a_1a_2a_2 \dots a_na_n).$$

Here, $s = 2^n$ is the number of different phenotypes and $x_i \in \{A_i, a_i\}$.

Theorem 5.2. [4] *Let* $H = \{\widehat{A_1}, \widehat{A_2}, \dots, \widehat{A_s}\}$. *Then* $(H, \otimes)$ *is a regular single power cyclic hypergroup.*

5.5.2 Genotypes of F_2-offspring

We consider the H_v-semigroup of genotypes in F_2-offspring discussed by the authors in details in [5] which is isomorphic to H_v-semigroup of phenotypes in F_2-

offspring under incomplete inheritance discussed in [4]. And the H_v-semigroup of phenotypes in F_2- offspring for the case of incomplete and simpleinheritance combined together.

The authors in [5] presented the results for the n-hybrid cross ($n \geq 1$) that differs in n traits; a homozygous parent $(A_1A_1A_2A_2 \ldots A_nA_n) \times$ a homozygous parent $(B_1B_1B_2B_2 \ldots B_nB_n)$. This experiment's results is summarized as follows.

$$P : A_1A_1A_2A_2 \ldots A_nA_n \times B_1B_1B_2B_2 \ldots B_nB_n$$
$$F_1 : A_1B_1A_2B_2 \ldots A_nB_n$$

and

$$F_1 \times F_1 : A_1B_1A_2B_2 \ldots A_nB_n \times A_1B_1A_2B_2 \ldots A_nB_n$$
$$F_2 : \widehat{B_1}, \widehat{B_2}, \ldots, \widehat{B_k},$$

where

$\widehat{B_1}$ represents $A_1A_1A_2A_2 \ldots A_nA_n$,

$\widehat{B_2}$ represents $A_1A_1A_2A_2 \ldots A_{n-1}A_{n-1}A_nB_n, \ldots,$

$\widehat{B_r}$ represents $A_1B_1A_2B_2 \ldots A_{n-1}B_{n-1}A_nB_n, \ldots,$

and $\widehat{B_s}$ represents $B_1B_1B_2B_2 \ldots B_nB_n$.

We have $s = 3^n$ different genotypes in F_2.

Proposition 5.3. [5] *Let $H = \{\widehat{B_1}, \widehat{B_2}, \ldots, \widehat{B_s}\}$ be the set of genotypes in F_2. Then $(H, \times)$ is a cyclic H_v-semigroup.*

The authors in [4] presented the results for the cross that differs in $m+n$ traits with $m, n \geq 1$ (The case of combination of incomplete and simple dominance); a homozygous parent $(B_1B_1 \ldots B_nB_n A_1A_1 \ldots A_mA_m) \otimes$ a homozygous parent $(\overline{B_1}\ \overline{B_1} \ldots \overline{B_n}\ \overline{B_n} a_1a_1 \ldots a_ma_m)$. The results of this hypothetical experiment can be summarized as follows.

$$P : B_1B_1 \ldots B_nB_nA_1A_1 \ldots A_mA_m \otimes \overline{B_1B_1} \ldots \overline{B_nB_n} a_1a_1 \ldots a_ma_m$$
$$F_1 : B_1\overline{B_1} \ldots B_n\overline{B_n}A_1a_1 \ldots A_ma_m$$

and

$$F_1 \otimes F_1 : B_1\overline{B_1} \ldots B_n\overline{B_n}A_1a_1 \ldots A_ma_m \otimes B_1\overline{B_1} \ldots B_n\overline{B_n}A_1a_1 \ldots A_ma_m$$

F_2 : $\widehat{A_1}$(of genotype $B_1B_1 \ldots B_nB_nA_1y_1 \ldots A_my_m$),

$\widehat{A_2}$(of genotype $B_1B_1 \ldots B_nB_nA_1y_1 \ldots A_{m-1}y_{m-1}a_ma_m$), $\ldots,$

$\widehat{A_r}$(of genotype $B_1B_1 \ldots B_nB_na_1a_1 \ldots a_ma_m$), $\ldots$

and $\widehat{A_s}$(of genotype $\overline{B_1B_1} \ldots \overline{B_nB_n}a_1a_1 \ldots a_ma_m$).

Here, $y_i \in \{A_i, a_i\}$ for $i = 1, \ldots, m$ and $s = 2^m 3^n$ is the number of different phenotypes.

Theorem 5.4. [4] *Let* $H = \{\widehat{A_1}, \widehat{A_2}, \ldots, \widehat{A_s}\}$. *Then* $(H, \otimes)$ *is a cyclic* H_v*-semigroup with identity and having* 2^n *idempotent elements.*

Here, $B_i, \overline{B_i}$ *are codominant alleles and* A_i *is dominant over the recessive allele* a_i.

5.6 Ordered Hyperstructures and Biological Ineritance

We present our results that relate ordered hyperstructures to biological inheritance. We work on the relation "$\leq$" that is defined as

$$x \leq y \Leftrightarrow x \in y^2.$$

The reason beyond choosing this relation is that if a genotype or a phenotype x appears in one of the offsprings that result from mating between two creatures sharing the same genotype or phenotype y then $x \leq y$ (x is weaker than y).

5.6.1 Phenotypes of F_2-offspring is a Po-hypergroup

Theorem 5.5. *Let* $(H, \otimes)$ *be the hypergroup defined in Example* 5.1. *Then* $(H, \otimes)$ *is an ordering hypergroup. Moreover,* $(H, \otimes, \leq)$ *is a po-hypergroup under the partial order relation defined in* [16].

Proof. From the table of $(H, \otimes)$ presented in Example 5.1, we can deduce the following:

$$A_1 \in A_1^2 = A_1^3, A_2 \in A_2^2 = A_2^3 \text{ and } A_3 \in A_3^2 = A_3^3,$$

$$A_1 \divideontimes A_2 = A_1^2 \cup A_2^2, A_1 \divideontimes A_3 = A_1^2 \cup A_3^2 \text{ and } A_2 \divideontimes A_3 = A_2^2 \cup A_3^2.$$

Thus, $(H, \otimes)$ is a quasi-ordering hypergroup.

To prove that $(H, \otimes)$ is an ordering hypergroup, let $x \neq y \in H$. We examine the following cases.

- Case $A_1 \neq A_2$. We have $A_1^2 = H \neq A_2^2 = \{A_2, A_3\}$.
- Case $A_1 \neq A_3$. We have $A_1^2 = H \neq A_3^2 = \{A_3\}$.
- Case $A_2 \neq A_3$. We have $A_2^2 = \{A_2, A_3\} \neq A_2^2 = \{A_3\}$.

Therefore, $(H, \otimes)$ is an ordering hypergroup.

Taking the partial order $\leq$ on H as:

$$x \leq y \Longleftrightarrow x \in y^2,$$

and using Theorem 2.11, we get that $(H, \otimes, \leq)$ is a po-hypergroup. □

Next, we consider the hypergroup of phenotypes in F_2- offspring under simple inheritance. Before presenting the main results of this subsection, we present the next proposition that is related to the monohybrid case ($n = 1$).

For the monohybrid case, $(H, \otimes)$ is presented by the following table.

$\otimes$	$\widehat{A_1}$	$\widehat{A_2}$
$\widehat{A_1}$	H	H
$\widehat{A_2}$	H	$\{\widehat{A_2}\}$

Proposition 5.6. $(H, \otimes, \leq)$ *in the monohybrid case is a po-hypergroup.*

Proof. Having $(H, \otimes)$ an ordering hypergroup, we get by Theorem 2.11 that $(H, \otimes, \leq)$ is a po-hypergroup. □

Theorem 5.7. $(H, \otimes, \leq)$ *in the n-hybrid case is a po-hypergroup.*

Proof. Let $\widehat{A_k}, \widehat{A_l} \in H$ be the phenotypes corresponding to the genotypes: $x_1x_1'x_2x_2' \dots x_nx_n'$ and $y_1y_1'y_2y_2' \dots y_ny_n'$ respectively. We have that $\widehat{A_k}^2$ is the set of phenotypes with the corresponding genotypes $z_1z_1'z_2z_2' \dots z_nz_n'$ where $\{z_i, z_i'\} = \{x_i, x_i'\}$. It is clear that $\widehat{A_k} \in \widehat{A_k}^2 = \widehat{A_k}^3$. Let $\widehat{A_l}^2$ and $\widehat{A_k} \otimes \widehat{A_l}$ be the set of phenotypes with the corresponding genotypes $t_1t_1't_2t_2' \dots t_nt_n'$ and $s_1s_1's_2s_2' \dots s_ns_n'$ respectively where $\{t_i, t_i', s_i, s_i'\} \subseteq \{x_i, x_i'\}$. Since each pair of x_i and x_i', y_i and y_i', z_i and z_i', t_i and t_i' are independent alleles for $i \in \{1, \dots, n\}$, we can consider just the i^{th} position while computing $\widehat{A_k} \otimes \widehat{A_l}, \widehat{A_k}^2, \widehat{A_l}^2$. We consider the following cases for the alleles of the i^{th} position.

- Case $x_i = A_i$ and $y_i = A_i$. We have that $z_iz_i' = t_it_i' = \{A_iA_i, A_ia_i, a_ia_i\} = s_is_i'$,
- Case $x_i = A_i$ and $y_i = a_i$.We have that $z_iz_i' = \{A_ix_i', x_i'x_i'\}$, $t_it_i' = a_ia_i$ and $s_is_i' = \{A_ix_i', x_i'x_i', a_ia_i\}$,
- Case $x_i = x_i' = a_i$ and $y_i = y_i' = a_i$. We have that $z_i = z_i' = a_i$ and $t_i = t_i' = a_i$. Moreover, $s_i = s_i' = a_i$.

The results of the above cases imply that $\widehat{A_k} \otimes \widehat{A_l} = \widehat{A_k}^2 \cup \widehat{A_l}^2$. We get now that $(H, \otimes)$ is a quasi-ordering hypergroup.

Let $\widehat{A_k} \neq \widehat{A_l}$ then there exists $i \in \{1, \dots, n\}$ such that $x_ix_i' \neq y_iy_i'$. We have only one case; $x_i = A_i$ and $y_i = a_i$. We get that $z_iz_i' = \{A_iA_i, A_ia_i, a_ia_i\}$ and $t_it_i' = a_ia_i$. Thus, $\widehat{A_k}^2 \neq \widehat{A_l}^2$.

Therefore, $(H, \otimes)$ is an ordering hypergroup. Theorem 2.11 completes the proof. □

Proposition 5.8. *Let $H = \{\widehat{A_1}, \widehat{A_2}, \dots, \widehat{A_s}\}$. Then the following properties hold:*

1. $(\widehat{A_r}] = H$,
2. $(\widehat{A_1}] = \widehat{A_1}$ *and* $(\widehat{A_s}] = \widehat{A_s}$,
3. $< \widehat{A_i} >= H$ *for all* $i = 1, \ldots, s$.

Proof. Since $\widehat{A_r}^2 = H$, $\widehat{A_1}^2 = \widehat{A_1}$ and $\widehat{A_s}^2 = \widehat{A_s}$, it follows that $(\widehat{A_r}] = \{t \in H : t \leq \widehat{A_r}\} = \{t \in H : t \in \widehat{A_r}^2\} = H$, $(\widehat{A_1}] = \widehat{A_1}$ and $(\widehat{A_s}] = \widehat{A_s}$.

Since $H \otimes \widehat{A_i} = H$, it follows that

$$< \widehat{A_i} >= (\widehat{A_i}] \cup (H \otimes \widehat{A_i}] = H.$$

$\square$

5.6.2 Genotypes of F_2-offspring is a Po- H_v- semigroup

We consider the H_v-semigroup of genotypes in F_2- offspring discussed by the authors in details in [5] which is isomorphic to H_v-semigroup of phenotypes in F_2-offspring under incomplete inheritance discussed in [4]. And the H_v-semigroup of phenotypes in F_2- offspring for the case of incomplete and simple inheritance combined together. First, we prove that our H_v-semigroups are not quasi-ordering. Then we define a partial order relation on them and prove that they are po-H_v-semigroups.

Proposition 5.9. *Let* $H = \{\widehat{B_1}, \widehat{B_2}, \ldots, \widehat{B_s}\}$ *be the set of genotypes in* F_2. *Then* $(H, \times)$ *is not quasi-ordering.*

Proof. Having $\widehat{B_1}^2 = \widehat{B_1}$, $\widehat{B_s}^2 = \widehat{B_s}$ and $\widehat{B_1} \times \widehat{B_s} = \widehat{B_r}$ implies that $\widehat{B_1} \times \widehat{B_s} \neq \widehat{B_1}^2 \cup \widehat{B_s}^2$. $\square$

Theorem 5.10. *Let* $H = \{\widehat{B_1}, \widehat{B_2}, \ldots, \widehat{B_s}\}$ *be the set of genotypes in* F_2 *and define the relation "*$\leq$*" on* H *as follows:*

$$x \leq y \Longleftrightarrow x \in y^2 \text{ for all } x, y \in H.$$

Then $\leq$ *is a poset on* H *satisfying the monotone property.*

Proof. Let $\widehat{B_k}, \widehat{B_l}$ be the genotypes $x_1x_1'x_2x_2' \ldots x_nx_n', y_1y_1'y_2y_2' \ldots y_ny_n'$ respectively, where $\{x_i, x_i', y_i, y_i'\} \subseteq \{A_i, B_i\}$ for all $i = 1, \ldots, n$.

We have that $\widehat{B_k}^2$ is the set of genotypes $z_1z_1'z_2z_2' \ldots z_nz_n'$ where $\{z_i, z_i'\} \subseteq \{x_i, x_i'\}$. It is clear that $\widehat{B_k} \in \widehat{B_k}^2$. Thus, $\widehat{B_k} \leq \widehat{B_k}$.

Let $\widehat{B_k} \leq \widehat{B_l}$ and $\widehat{B_l} \leq \widehat{B_k}$. Then $\widehat{B_k} \in \widehat{B_l}^2$ and $\widehat{B_l} \in \widehat{B_k}^2$. Since each pair of x_i and x_i', y_i and y_i' are independent alleles for all $i = 1, \ldots, n$, we can consider just the i^{th} position. The latter implies that $x_ix_i' \in \{y_iy_i, y_iy_i', y_i'y_i'\}$ and $y_iy_i' \in \{x_ix_i, x_ix_i', x_i'x_i'\}$. We get the following cases for the alleles of the i^{th} position:

- Case $x_i = x'_i = y_i$. We get $y_iy'_i \in \{x_ix'_i\} = \{y_iy_i\}$. Thus, $x_ix'_i = y_iy'_i$,
- Case $x_i = y_i$ and $x'_i = y'_i$. It is clear that $x_ix'_i = y_iy'_i$,
- Case $x_i = x'_i = y'_i$. We get $y_iy'_i \in \{y'_iy'_i\}$. Thus, $y_i = y'_i = x_i = x'_i$.

To prove transitivity, let $\widehat{B_m} = u_1u'_1u_2u'_2 \dots u_nu'_n \in H$ such that $\widehat{B_k} \leq \widehat{B_l}$ and $\widehat{B_l} \leq \widehat{B_m}$. Using the same argument as before, we get that $x_ix'_i \in \{y_iy_i, y_iy'_i, y'_iy'_i\}$ and $y_iy'_i \in \{u_iu_i, u_iu'_i, u'_iu'_i\}$. We consider the following cases for $y_iy'_i$:

- Case $y_iy'_i = u_iu_i$. Then $y_i = y'_i = u_i$ and thus, $x_ix'_i \in \{u_iu_i\}$.
- Case $y_iy'_i = u_iu'_i$. Then $y_i = u_i$ and $y'_iu'_i$. Thus, $x_ix'_i \in y_iy'_i \in \{u_iu_i, u_iu'_i, u'_iu'_i\}$ $x_ix'_i \in \{u_iu_i\}$.
- Case $y_iy'_i = u'_iu'_i$. Then $y_i = y'_i = u'_i$ and thus, $x_ix'_i \in \{u_i\prime u_i\prime\}$.

We get now that $\leq$ is transitive. Therefore, $\leq$ is a poset.

We need to show that $\leq$ satisfies the monotone property. Suppose that $\alpha = s_1s'_1s_2s'_2 \dots s_ns'_n \in H$, $\widehat{B_k} \leq \widehat{B_l}$ and $\beta = m_1m'_1m_2m'_2 \dots m_nm'_n \in \alpha \times \widehat{B_k}$. Having $\beta \in \alpha \times \widehat{B_k}$ implies that $m_im'_i \in \{s_ix_i, s_ix'_i, x_is'_i, s'_ix'_i\}$. We get the following cases for the alleles of the i^{th} position of β: $m_im'_i = s_ix_i$, $m_im'_i = s_ix'_i$, $m_im'_i = x_is'_i$ and $m_im'_i = x'_is'_i$. We prove the case $m_im'_i = s_ix_i$ and the others are done in a similar manner.

We have that the alleles of the i-th position in $\alpha \times \widehat{B_l}$ are $\{s_iy_i, s'_iy_i, s_iy'_i, s'_iy'_i\}$ and that of $(\alpha \times \widehat{B_l})^2 = \{\gamma^2; \gamma \in \alpha \times \widehat{B_l}\}$ are

$$\{y_iy_i, s_is_i, y_iy'_i, y'_iy'_i, s_iy_i, s_iy'_i, s_is'_i, s'_iy_i, s'_iy'_i, s'_is'_i\}.$$

Since $\widehat{B_k} \in \widehat{B_l}^2$, it follows that $x_ix'_i \in \{y_iy_i, y_iy'_i, y'_iy'_i\}$ and thus, $m_im'_i \in \{s_iy_i, s_iy'_i\}$. Therefore, $\alpha \times \widehat{B_k} \leq \alpha \times \widehat{B_l}$. □

Corollary 5.11. *Let $H = \{\widehat{B_1}, \widehat{B_2}, \dots, \widehat{B_s}\}$ be the set of genotypes in F_2. Then $(H, \times, \leq)$ is a po-H_v-semigroup.*

Proof. The proof results from Theorems 2.11 and 5.10. □

Remark 4. A po-H_v-semigroup needs not to be a quasi ordering H_v-semigroup. This is illustrated by Proposition 5.9 and Corollary 5.11.

Proposition 5.12. *Let $H = \{\widehat{B_1}, \widehat{B_2}, \dots, \widehat{B_s}\}$. Then the following properties hold:*

1. $(\widehat{B_r}] = H$,
2. $(\widehat{B_1}] = \widehat{B_1}$ *and* $(\widehat{B_s}] = \widehat{B_s}$,
3. $< \widehat{B_i} >= H$ *for all* $i = 1, \dots, s$.

Proof. Since $\widehat{B_r}^2 = H$, $\widehat{B_1}^2 = \widehat{B_1}$ and $\widehat{B_s}^2 = \widehat{B_s}$, it follows that $(\widehat{B_r}] = \{t \in H : t \leq \widehat{B_r}\} = \{t \in H : t \in \widehat{B_r}^2\} = H$, $(\widehat{B_1}] = \widehat{B_1}$ and $(\widehat{B_s}] = \widehat{B_s}$.

Since $\widehat{B_r} \in \widehat{B_r} \otimes \widehat{B_i}$, it follows that

$$< \widehat{B_i} >= (\widehat{B_i}] \cup (H \otimes \widehat{B_i}] = H.$$

□

The authors in [4] presented the results for the cross that differs in $m+n$ traits with $m.n \geq 1$ (The case of combination of incomplete and simpledominance); a homozygous parent $(B_1B_1 \ldots B_nB_nA_1A_1 \ldots A_mA_m) \otimes$ a homozygous parent $(\overline{B_1}\ \overline{B_1} \ldots \overline{B_n}\ \overline{B_n}a_1a_1 \ldots a_ma_m)$. where $s = 2^m3^n$ is the number of different phenotypes and $y_i = A_i$ or a_i for $i = 1, \ldots, m$,

Proposition 5.13. *Let $H = \{\widehat{A_1}, \widehat{A_2}, \ldots, \widehat{A_s}\}$ and $s = 2^m3^n$. Then $(H, \otimes)$ is not quasi-ordering.*

Proof. Having
$\widehat{A_s}^2 = \widehat{A_s}$, $\widehat{A_r}^2 = \widehat{A_r}$ and $\widehat{A_k} \otimes \widehat{A_r} = \widehat{A_s}$ (of genotype $B_1\overline{B_1} \ldots B_n\overline{B_n}a_1a_1 \ldots a_ma_m$) implies that $\widehat{A_k} \times \widehat{A_r} \neq \widehat{A_k}^2 \cup \widehat{A_r}^2$. □

Theorem 5.14. *Let $H = \{\widehat{A_1}, \widehat{A_2}, \ldots, \widehat{A_s}\}$ and define a relation $\leq$ on H as follows:*

$$x \leq y \Longleftrightarrow x \in y^2 \text{ for all } x, y \in H.$$

Then $\leq$ is a poset on H satisfying the monotone property.

Proof. The proof is similar to that of Theorem 5.10. □

Corollary 5.15. *Let $H = \{\widehat{A_1}, \widehat{A_2}, \ldots, \widehat{A_s}\}$. Then $(H, \otimes, \leq)$ is a po-H_v-semigroup.*

Proof. The proof is a result of Theorems 2.11 and 5.14. □

6. Conclusion

This manuscript discusses quasi-ordering hypergroups by studying its properties and determining its fundamental group and complete parts. Also, it improved the authors' work in [4, 5] by studying the ordered hypercompositional structures of both phenotypes and genotypes of F_2-offspring.

Acknowledgment. The authors thank the referees for the constructive comments.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this article.

References

[1] M. Al- Tahan and B. Davvaz, On a special single-power cyclic hypergroup and its automorphisms, *Discrete Math. Algorithms Appl.* **8** (4) (2016) 1650059, 12 pp.

[2] M. Al- Tahan and B. Davvaz, On some properties of single-power cyclic hypergroups and regular relations, *J. Algebra Appl.* **16** (11) (2017) 14 pp.

[3] M. Al- Tahan and B. Davvaz, Commutative single-power cyclic hypergroups of order three and period two, *Discrete Math. Algorithms Appl.* **19** (5) (2017) 1750070, 15 pp.

[4] M. Al- Tahan and B. Davvaz, Algebraic hyperstructures associated to biological inheritance, *Math. Biosci.* **285** (2017) $112-118$.

[5] M. Al- Tahan and B. Davvaz, N-ary hyperstructures associated to the genotypes of F_2-offspring, *Int. J. Biomath.* **10** (8) (2017) 1750118, 17 pages.

[6] M. Bakhshi and R. A. Borzooei, Ordered polygroups, *Ratio Math.* **24** (2013) $31-40$.

[7] J. Chvalina, Commutative hypergroups in the sense of Marty and ordered sets, *Gen. Alg. and Ordered Sets, Proc. Inter. Conf., Olomouc* (1994) $19-30$.

[8] J. Chvalina and J. Moucka, Hypergroups determined by ordering with regular endomorphism monoids, *Ital. J. Pure Appl. Math.* **16** (2004) $227-242$.

[9] P. Corsini, *Prolegomena of Hypergroup Theory*, 2nd ed., Aviani Editore, Tricesimo, Italy, 1993.

[10] P. Corsini and V. Leoreanu, *Applications of hyperstructures theory*, Advances in Mathematics, Kluwer Academic Publisher, Dordrecht, 2003.

[11] B. Davvaz, A. D. Nezhad and M. Heidari, Inheritance examples of algebraic hyperstructures, *Inform. Sci.* **224** (2013) $180-187$.

[12] B. Davvaz, *Polygroup Theory and Related Systems*, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, USA, 2013.

[13] B. Davvaz and T. Vougiouklis, *A Walk Through Weak Hyperstructures H_v-Structures*, World Scientific: Hackensack, NJ, USA, 2018.

[14] M. De Salvo and D. Freni, Cyclic semihypergroups and hypergroups, (Italian) *Atti Sem. Mat. Fis. Univ. Modena* **30** (1) (1981) $44-59$.

[15] D. Freni, A note on the core of a hypergroup and the transitive closure β^* of β, *Riv. Mat. Pura Appl.* **8** (1991) $153-156$.

[16] D. Heidari and B. Davvaz, On ordered hyperstructures, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys. **73** (2) (2011) 85 − 96.

[17] D. A. Hort, A construction of hypergroups from ordered structures and their morphisms, *J. Discrete Math. Sci. Cryptogr.* **6** (2003) 139 − 150.

[18] Š. Hošková, Representation of quasi-order hypergroups, *Glob. J. Pure Appl. Math.* **1** (2005) 173 − 176.

[19] M. Koskas, Groupoides, demi-hypergroupes et hypergroupes (French), *J. Math. Pure Appl.* **9** (49) (1970) 155 − 192.

[20] V. Leoreanu, About the simplifiable cyclic semihypergroups, *Ital. J. Pure Appl. Math.* **7** (2000) 69 − 76.

[21] V. Leoreanu, Commutative hypergroups associated with arbitrary hypergroups, *J. Discrete Math. Sci. Cryptogr.* **6** (2-3) (2003) 195 − 198.

[22] F. Marty, Sur une generalization de la notion de group, *in: 8th Congress Math. Scandenaves, Stockholm, Sweden* (1934) 45 − 49.

[23] C. Massouros and G. Massouros, Transposition hypergroups with idempotent identity, *Int. J. Alg. Hyperstructures Appl.* **1** (2014) 15 − 27.

[24] G. Mendel, Versuche über Pflanzen-Hybriden, *Verh. Naturforsch. Ver. Brünn* **4** (1866) 3 − 47 (in English in: *J. R. Hortic. Soc.* **26** (1901) 1 − 32.

[25] M. Novak, On EL-semihypergroups, *Eur. J. Combin.* **44** (2015) 274 − 286.

[26] M. Novak, Some basic properties of EL-hyperstructures, *Eur. J. Combin.* **34** (2013) 446 − 459.

[27] S. Omidi and B. Davvaz, Foundations of ordered (semi)hyperrings, *J. Indones. Math. Soc.* **22** (2) (2016) 131 − 150.

[28] I. G. Rosenberg, Hypergroups and join spaces determined by relations, *Ital. J. Pure Appl. Math.* **4** (1998) 93 − 101.

[29] T. Vougiouklis, *Hyperstructures and Their Representations*, Aviani editor. Hadronic Press, Palm Harbor, USA, 1994.

[30] T. Vougiouklis, Cyclicity in a special class of hypergroups, *Acta Univ. Carolin. Math. Phys.* **22** (1) (1981) 3 − 6.

Madeleine Al- Tahan
Department of Mathematics,
Lebanese International University,
Lebanon
e-mail: altahan.madeleine@gmail.com

Bijan Davvaz
Department of Mathematics,
Yazd University,
Yazd, Iran
e-mail: davvaz@yazd.ac.ir