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Coupling Chebyshev Collocation with TLBO to Optimal Control Problem of Reservoir Sedimentation: A Case Study on Golestan Dam, Gonbad Kavous City, Iran

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Abstract

In this paper, an efficient and robust approach based on the Chebyshev collocation method and Teaching-Learning-Based Optimization (TLBO) is utilized to solve the Optimal Control Problem (OCP) of reservoir sedimentation on Golestan dam in Gonbad Kavous City, Iran. The discretized method employs Mth degree of Lagrange polynomial approximation for an unknown variable and Gauss-Legendre integration. The OCP yields a nonlinear programming problem (NLP), and then this NLP is solved by TLBO. Numerical implementations are given to demonstrate this approach yields more acceptable and the accurate results. Furthermore, it is found that filling the dam with sediment decreases the water storage, increases dam maintenance costs, and also decreases the stability of the dam over a period of 40 years. Our results show that the Golestan dam will gain development with the construction of the new reservoir.

Keywords: Optimal control, Reservoir sedimentation, Collocation method, Teaching learning based optimization, Golestan dam.

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1. Introduction

About 1% of the storage volume of reservoirs is filled due to sedimentation all over the world. Sedimentation in reservoirs of dams is inevitable and the useful life of dams is defined based on an estimate of the time required to fill them due to sedimentation. The important point to be considered while taking the operation of dams is the failure to take the operation of the dam due to the sediment entering it. Golestan province is one of the agricultural poles, and many dams have been built on its water sources. The occurrence of floods has necessitated a more detailed investigation of the sediment status in dam reservoirs and strategies have been provided in order to use them more appropriately.

With the construction of the dam on the river, a lake is created above the dam, which reduces the velocity of water flow and the carrying capacity of the river. This causes the sedimentation in reservoirs of the dam. Sedimentation in the reservoirs of the dam is inevitable. Therefore, the deposition will have the following adverse effects:

1. Severe reduction of the volume for dam reservoirs.
2. Creating the adverse effects on hydropower generation.
3. Creating problems in navigation and water production.
4. Decreasing the stability of the dam and increasing the force on the dam body.
5. Having a negative impact on the water outlet facilities and dam outlet valves.
6. Increased evaporation in a large area of reservoirs.
7. Decreasing the quantity of water consumption and reducing the capacity of flood control volume and recreational benefits.

Therefore, it is necessary to study the sedimentation in the planning, design, operation, maintenance and repair of dams, and reservoirs. Many researchers, around the world, have studied the OCP of sediment management in reservoir's dams. In [1], an optimization methodology was utilized that transformed the constrained problem in to an unconstrained one using the penalty function and the resulting problem was solved by the Powell method. In [2], a successive approximation linear quadratic regulator approach had applied to dissolve the introduced OCP for minimizing the sedimentation in rivers and reservoirs. In [3], an OCP was presented to minimize the total cost and determine the optimal location in navigation channels. The OCP was solved by the conjugate gradient method. Also, a simulation-based optimal control model was presented and an LMQN optimization procedure is adopted for obtaining the optimal flood control actions in [4]. In [5], a multi-dimensional OCP governed by singular perturbed equations had modelled. The singular perturbation method had been utilized to transform the multi-dimensional solution region to the single-dimensional sub-region. In [6], an ecological well passed control problem of the partial differential equations had modelled and dissolved by a numerical algorithm to manage sedimentation in reservoirs. In [7], a mathematical model was demonstrated to study about the optimal of flood stage for different geometric and sediment characteristics. The related researches based on the problem and network type, and numerical and optimization methods are reported in Table 1. Surely, Table 1 shows a classification of the concerned works. In Table 1, the mark "✓" shows that the characteristic exists in the literature, while the mark "×" demonstrates that it does not exist. As stated in the above literature, OCPs are modelled to determine the various sedimentation management strategies. Till now, the various approaches are utilized to transform the OCP to NLP to earn approximate control and to state the functions that under certain conditions converge to the exact one, and then suitable optimization technique should be applied to obtain an optimal solution with a high accuracy and short time. So, an efficient metaheuristic method will be executed based on TLBO for the OCP of reservoir sedimentation. It is noteworthy that, TLBO is utilized as one of the powerful metaheuristics to solve the large-scale NLPs (see [8–11]). Among all of the metaheuristic optimization techniques, the TLBO is selected for the OCP of reservoir sedimentation that deals with determining control variable of remaining storage capacity, the volume of impounded water and consumption rate of storage water in non-hydrosuction activities due to the small number of control parameters and suitable performance concerning accuracy and CPU time in comparison with the other metaheuristics.

This study present a direct collocation method for transforming OCP to NLP. The advantages of using this method which is described in Section 3 with the details are as follows:

1. By using Chebyshev collocation points and Lagrange polynomials, the OCP is readily transformed to NLP;
2. This approach can easily be applied to a wide range of OCPs, maybe with very minor changes despondingly in the form of the system equations;
3. The unknown variables are obtained directly at the collocation points;
4. The easiness and capability of this Chebyshev collocation method allows one to do swift and the exact optimizations;
5. A low degree of polynomial appears to give the acceptable and satisfactory results.

In recent years, considerable attention has been devoted to the use of collocation method to solve OCPs. Due to its generality, simple implementation, computational efficiency in large-scale problems and high accuracy, this method has been executed to solve many OCPs in various fields. By using the collocation method,

Table 1: A classification of the available problems and methods in the literature.

Papers	Problem type	Numerical approach	Optimization method	Network	Case study
Valizadegan et al. [1]	OCP	GSTARS	Powell method	Sedimentation	✓
Nicklow and Mays [2]	OCP	×	Successive approximation linear quadratic regulator	Sedimentation	✓
Zhu et al. [3]	OCP	Finite-difference	Conjugate gradient	Sedimentation	✓
Ding et al. [4]	OCP	Preissmann scheme	Limited-memory quasi newton	Sedimentation	✓
Huffaker and Hotchkiss [5]	OCP governed by singular perturbed equation	Singular perturbation	×	Sedimentation	✓
Alvarez-Vázquez et al. [6]	OCP	×	Nelder Mead algorithm	Sedimentation	✓
Ding and Wang [7]	OCP	implicit four point finite difference	BFGS algorithm	Sedimentation	✓
Khanduzi et al. [12]	OCP	Chebyshev wavelet	TLBO	General	×
Kundu et al. [13]	Structural damage detection	×	TLBO	Structural	✓
Hays et al. [14]	Ordinary differential equation	Least square collocation	Sequential and simultaneous nonlinear programming	Parametric design	✓
Valian et al. [15]	OCP	Fractional-order Bernoulli wavelet	×	General	×
Lee et al. [16]	Convection-diffusion equation	NETSTARS	Split-Operator	Sedimentation	✓
Chen et al. [17]	OCP	Control vector parameterization	TLBO	Chemical	✓
Kaveh et al. [18]	Time-independent Schrodinger differential equation	×	TLBO	Structural	✓
Current study	OCP governed by singular perturbed equation	Chebyshev collocation	TLBO	Sedimentation	✓

the OCP is converted to a NLP. Considering that the NLP is discrete and large-scale, it is possible to solve the problem with numerical optimization algorithms accurately in a long time, and only the local optimal solution of the problem is possible. Therefore, due to the good performance of metaheuristic algorithms in achieving high-quality solutions in a short time, TLBO is presented to solve this problem.

Sedimentation in reservoirs of dams is always one of the major problems, and numerous challenges for water resources management, and control of this phenomenon has a lot of importance. This phenomenon reduces reservoir capacity, reducing dam stability, decreasing flood control volume, improper performance of exploitation hatchways and formation of sedimentary islands at the entrance of the river to reservoir. In this work, an optimal control approach for minimization of sediment in reservoirs is studied and the OCP is converted to a constrained nonlinear optimization problem using collocation method and approximation of the unknown variables with appropriate basic functions and using the operating matrix of those functions. Then, due to the nonlinear nature and large dimensions of this problem, a metaheuristic algorithm, i.e., TLBO is presented with the appropriate structure and the results of the algorithm are presented. A case study is considered to demonstrate the proper performance of discretization approach and the efficiency of metaheuristic algorithms. The following are the main contributions and advantages of this study:

- A collocation approach is proposed to convert the OCP model into a finite dimensional mathematical programming problem.
- The derivative operational matrices is sparse, so the proposed approach is easy to implement.
- As far as we know, TLBO has not been executed for solving NLP governed by OCP of reservoir sedimentation in the literature.
- For the first time in the literature, the proposed method is used to solve the optimal control with singularly perturbed equations of the problem related to reservoir sedimentation.

This work is outlined as follows: In Section 2, a detailed mathematical model of OCP is described. In Section 3, a solution methodology for solving this problem is presented. In Section 4, the case study of OCP of the reservoir sedimentation and numerical results will be presented. Section 5 summarizes our conclusions.

2. Model description

In this section, mathematical modeling of optimal control with singularly perturbed equations of the problem related to reservoir sedimentation is proposed. According to the mathematical model presented, the parameters, variables and functions are indicated in Tables 2 to 4 .The information given in Table 2 is related to the data of Golestan dam in Gonbad Kavous city in Iran that is taken from the Regional Water Company of Golestan (RWCG). The following entities that are inserted in Tables 3 and 4 are respectively the variables and functions utilized in the OCP of sedimentation in the reservoir. Notations utilized in this optimization model are given below: $e(w(t), s(t))$ is the performance of experi-

Table 2: The value of data in economic sediment-transport model

Parameter	Description	Value	Unit
r	Discount factor	784	$1/year$
s_0	Initial storage volume	86000000	m^3
L	Evaporation factor	1173/3	$1/year$
α	Composite discount rate	1958/3	$1/year$
om	Operating and maintenance costs	4500000000	IRR/Year
η	Mean annual sediment flux	2060000	$m^3/Year$
γ	Ratio of water required to eliminate a unit of sediment	0	-
p_h	Unit net profit from eliminating sediment	0	IRR/Year
p_c	Unit net profit from water consumption	30000000000	IRR/Year
e^m	Maximum reservoir height	17	m
$F_1(F_2)$	Slope (intercept) of sediment transport-height	$15/74(5*10^{-15})$	$m^3/Year/m$
ϕ_1	Composite constant	267.59	-
ϕ_2	Composite constant	$13*10^{-11}$	-
k_1	Height-storage water performance score	0.08	-
$c_{max}(c_{min})$	Maximum (minimum) consumption factor	1550 (0)	$m^3/Year$
ε	Perturbation constant	0.07	-

Table 3: Variables of the economic sediment-transport model

Variables	Description	Unit
$s(t)$	Remaining storage volume	m^3
$w(t)$	The amount of impounded water	m^3
$c(t)$	Consumption amount of storage water in non-hydrosuction process	$m^3/Year$

Table 4: Functions

Functions	Description	Unit
$e(w,s)$	Height-storage	m
$x(e)$	Sediment transport-height	$m^3/Year$
$R(w,s)$	Reservoir refill operating	$m^3/Year$
$y(x)$	Water extracted from hydrosuction pipeline	$m^3/Year$
$ev(w)$	Evaporative water loss	$m^3/Year$
$D(s)$	Sediment settling	m^3

mental elevation-storage curves which is modelled as a Michaelis-Menton equation:

$$e(w(t), s(t)) = \frac{e^{max} w(t)}{w(t) + D(s(t))}, \quad D(s(t)) = k_1 s(t). \tag{1}$$

The transport-height function, i.e., $x(e(t))$ is computed as:

$$x(e(t)) = F_1 e(t) + F_2. \tag{2}$$

Replacing Equation (1) into (2) modifies the sediment transport factor by considering a mathematical relationship of the two-state variables ($w(t)$ and $s(t)$):

$$x(w(t), s(t)) = \frac{\phi_1 w(t)}{w(t) + D(s(t))} + F_2, \quad \phi_1 = e^{max} F_1. \tag{3}$$

Water is extracted from hydro-suction pipeline, i.e., $y(x(t))$, is directly proportional to the sediment transport factor:

$$y(x(t)) = \gamma x(w(t), s(t)). \tag{4}$$

The reservoir has a storage region for annual extra water, $R(w(t), s(t))$, corresponding to the difference between the resting storage volume and amount of water impounded:

$$R(w(t), s(t)) = s(t) - w(t). \tag{5}$$

Evaporative losses for each year, $ev(w(t))$, are estimated as a fixed fraction of impounded water:

$$ev(w(t)) = Lw(t). \tag{6}$$

Also, time-dependent changes of impounded water and storage volume are defined by:

$$\frac{dw}{dt} = R(w(t), s(t)) - c(t) - y(x(t)) - ev(w(t)), \tag{7}$$

$$\frac{ds}{dt} = \varepsilon(x(w(t), s(t)) - \eta). \tag{8}$$

In Equation (7), the amount of impounded water varies at a net factor during a year correspond to the difference between refill for each year and the consumptive activities. These involve the factors of impounded-water utilization in non-hydrosuction activities, impounded-water utilization in hydrosuction dredging, and evaporative losses. In Equation (8), the annual net factor of change in storage volume is the difference between the sediment transport factor and a constant factor for each year at which sediment is involved in the reservoir. Equations (7) and (8) comprise a structure of singularly perturbed differential equations. The flow of net revenues for each year from the dam/reservoir scheme is:

$$p_c c(t) + p_h x(w(t), s(t)) - om. \tag{9}$$

Here, $p_c c(t)$ shows the net revenue of each year from the utilization of impounded water in non-hydro-suction activities and $p_h x(w(t), s(t))$ shows the annual net revenue from sediment transport.

2.1. Mathematical model

The mathematical model of the OCP for reservoir sedimentation management is represented as follows:

$$J = \max_{c(t)} \int_0^T e^{-rt}(p_c c(t) + p_h x(w(t), s(t)) - om)dt + e^{-rT}V, \qquad (10)$$

$$\frac{dw}{dt} = R(w(t), s(t)) - c(t) - \gamma x(w(t), s(t)) - ev(w(t)), \quad w(0) = w_0, \qquad (11)$$

$$\frac{ds}{dt} = \epsilon(x(w(t), s(t)) - \eta), \quad s(0) = s_0, \qquad (12)$$

$$c_{min} \leq c(t) \leq c_{max}, \qquad (13)$$

where w_0 and s_0 are the primary requirements on the state variables. Here, T is the dam lifespan, and $e^{-rT}V$ is a discounted salvage value relating to the designer's decision at the dam's loss. This problem is a rapid approach problem as considered in [19]. It needs the upper and lower limits on the control variable: $c_{min} \leq c(t) \leq c_{max}$.

3. Solution Methodology

In this section, we focus on a numerical-optimization approach to solve the OCP of sedimentation in the reservoir.

3.1. Collocation method

In this section, the essential mechanism of the collocation method is explained. Let T_i be the ith order Chebyshev polynomials which are defined by:

$$T_i(\tau) = \cos(i \cos^{-1} \tau), \quad -1 \leq \tau \leq 1, \quad i = 0, ..., M,$$

and

$$t_l = cos\left(\frac{\pi l}{M}\right), \quad l = 0, 1, \cdots, M, \qquad (14)$$

are the extremes of Mth order Chebyshev polynomial. Since the problem presented in subsection 2.1 is given in the interval $[0, T]$, and the points t_l lie in the interval $[-1, 1]$, we use the following transformation to express the problem in interval $[-1, 1]$:

$$t = \frac{T}{2}(\tau + 1). \qquad (15)$$

The Lagrange polynomials of order M, for $i = 0, 1, ..., M$ are defined as:

$$\phi_i(\tau) = \frac{(-1)^{i+1}(1-\tau^2)T_M^{'}(\tau)}{M^2\sigma_i(\tau - t_i)}, \tag{16}$$

where σ_i is

$$\sigma_i = \begin{cases} 2, & i = 0, M, \\ 1, & 1 \leq i \leq M-1. \end{cases}$$

It can be demonstrated that

$$\phi_i(t_l) = \delta_{il} = \begin{cases} 1, & i = l, \\ 0, & i \neq l. \end{cases} \tag{17}$$

For approximating the unknown functions, we utilize a polynomial approximation of the form:

$$s^M(\tau) = \sum_{i=0}^{M} s_i\phi_i(\tau) = S^T\phi(\tau), \tag{18}$$

$$w^M(\tau) = \sum_{i=0}^{M} w_i\phi_i(\tau) = W^T\phi(\tau), \tag{19}$$

$$c^M(\tau) = \sum_{i=0}^{M} c_i\phi_i(\tau) = C^T\phi(\tau), \tag{20}$$

where

$$S^T = [s_0, s_1, ..., s_M], \quad W^T = [w_0, w_1, ..., w_M], \quad C^T = [c_0, c_1, ..., c_M],$$

and

$$\phi(\tau) = [\phi_0(\tau), \phi_1(\tau), ..., \phi_M(\tau)]^T.$$

Note that, from Equation (17) we have

$$s^M(t_l) = s_l, \quad w^M(t_l) = w_l, \quad c^M(t_l) = c_l, \quad l = 0, 1, \cdots, M.$$

To approximate $w'(\tau)$ and $s'(\tau)$ in collocation points t_l, we differentiate $w^M(\tau) = \sum_{j=0}^{M} w_j\phi_j(\tau)$ and $s^M(\tau) = \sum_{j=0}^{M} s_j\phi_j(\tau)$ and substirute τ by t_l, so we have

$$w'^M(t_l) = \sum_{j=0}^{M} d_{lj}w_j, \qquad s'^M(t_l) = \sum_{j=0}^{M} d_{lj}s_j. \tag{21}$$

Where d_{lj} are the entries of the operational matrix of derivatives D at point t_l given by [20]

$$d_{lj} = \begin{cases} \frac{\sigma_l(-1)^{l+j}}{\sigma_j(t_l - t_j)}, & l \neq j, \\ \frac{-t_l}{2(1-t_l^2)}, & 1 \leq l = j \leq M-1, \\ \frac{2M^2+1}{6}, & l = j = 0, \\ -\frac{2M^2+1}{6}, & l = j = M. \end{cases} \tag{22}$$

We substituted Equations (18)-(20) into constraints (11)-(13). Then, we collocated the resulting equations at points t_l given in Equation (14), which allowed us to obtain the following equations for $l = 0, 1, 2, \cdots, M$:

$$-\frac{2}{T}(\sum_{j=0}^{M} d_{lj} w_j) + R(w_l, s_l) - c_l - \gamma x(w_l, s_l) - ev(w_l) = 0, \tag{23}$$

$$-\frac{2}{T}(\sum_{j=0}^{M} d_{lj} s_j) + \varepsilon(x(w_l, s_l) - \eta) = 0,$$

$$c_{min} \leq c_l \leq c_{max}.$$

After transforming the objective functional given in Equation (10) appropriately, we discretized it using Equations (18)-(20) and numerical integration, which is resulted in the following equation

$$\frac{T}{2}\sum_{i=0}^{M} \omega_i (e^{-rq_i})(p_c C^T \phi(q_i) - p_h x(W^T \phi(q_i), S^T \phi(q_i)) - om)) + e^{-rT} V, \tag{24}$$

where q_i is the Gauss-Legendre node, zeros of Legendre polynomials $\mathcal{P}_{M+1}$, and ω_i is the corresponding weight. The quadrature weights, ω_i , are given by following formula [15]:

$$\omega_i = \frac{2}{(1 - (q_i^2))(\mathcal{P}'_{M+1}(q_i))^2}, \qquad i = 0, 1, 2, ..., M.$$

3.2. Teaching-Learning-Based Optimization

This subsection describes the TLBO and its elements and mechanisms to solve NLP governed by OCP. The TLBO was a metaheuristic algorithm proposed in [10] as stated by the impact grade of a teacher on the teaching of students in a classroom. The method mimicked the training-studying principle. Teacher and students were the two central elements of the method and it was explained by two mechanisms, i.e., teacher and students step. The result of TLBO method was described by the score of the students regarding the condition of teacher education. A good teacher teaches the students so that they can acquire better grades. On the other hand, students acquired knowledge from cooperation with other students, which increased their scores. Also, a classroom of students was studied as population and various issues to the students were suggested as the various parameters of the objective value. The best student (minimum objective function) in the classroom was assumed as the teacher. This algorithm comprised two steps, teacher step and student step that were described as follows:

3.2.1. Teacher step

In this step, the teacher trains the students and attempts to improve the average score of the classroom based on his or her ability. At any iteration i, there are the inputs and parameters as follows:

u	Number of issues
v	Number of students
$A_{j,i}$	The average outcome of the students in a special issue $j(j = 1, ..., u)$
$Z_{t-s_{best},i}$	The best overall outcomes
s_{best}	The answer of best student
$Z_{j-s_{best},i}$	The answer of the best student (i.e., teacher) in subject j
n_i	The random number in the range $[0, 1]$

The teacher (the best student person) teaches students with the aim that they can acquire better scores. The difference between the average outcome of each issue and the teacher outcome for each issue is determined by:

$$D - A_{j,k,i} = n_i(Z_{j-s_{best},i} - E_F A_{j,i}). \tag{25}$$

Where E_F is the educating rate which chooses the value of the average to be varied that is selected by:

$$E_F = round[1 + rand(0, 1)2 - 1]. \tag{26}$$

In this step, the solution changed regarding the $D - A_{j,k,i}$ based on the below equation:

$$Z^{'}_{j,k,i} = Z_{j,k,i} + D - A_{j,k,i}. \tag{27}$$

Where $Z^{'}_{j,k,i}$ is the new value of $Z_{j,k,i}$. $Z^{'}_{j,k,i}$ is confirmed if it obtains a better objective value. Also, all the confirmed objective values are considered as the input to the student step.

3.2.2. Student step

In this step of the TLBO, students improve their skills by cooperating with other students. A student cooperates randomly with the other students to improve their skills. Thus, a student acquires new knowledge if the other student has more skill than his or her, the studying law is modelled as follows.

Randomly select the two students X and Y such that $Z^{'}_{t-X,i} \neq Z^{'}_{t-Y,i}$ where, $Z^{'}_{t-X,i}$ and $Z^{'}_{t-Y,i}$ are the new values of $Z_{t-X,i}$ and $Z_{t-Y,i}$ respectively, at teacher step.

$$Z^{''}_{j,X,i} = Z^{'}_{j,X,i} + n_i(Z^{'}_{j,X,i} - Z^{'}_{j,Y,i}), \text{ if } Z^{'}_{j,X,i} < Z^{'}_{j,Y,i}, \tag{28}$$

$$Z^{''}_{j,X,i} = Z^{'}_{j,X,i} + n_i(Z^{'}_{j,Y,i} - Z^{'}_{j,X,i}), \text{ if } Z^{'}_{j,Y,i} < Z^{'}_{j,X,i}. \tag{29}$$

Confirm $Z^{''}_{j,X,i}$, if it obtains a better solution. All the confirmed solutions at student step are considered as the input to the next repetition. The values of in relations (25), (28) and (29) can be different. Replicate the teacher step and student step until the condition is provided. TLBO algorithm can be executed to solve the OCP as follows:

Step 1: Initialize the parameters, the population, variables and stopping condition.

Step 2: Generate the initial population.

Step 3: Compute the average of each variable.

Step 4: Choose the best answer.

Step 5: Compute the $D - A$ and change the answers regarding the best answer.

Step 6: If new answer is better than the existing, choose the answers randomly and change them by comparing with each other, otherwise, maintain the previous answer.

Step 7: If new answer is better than existing answer, go to step 8, otherwise, maintain the previous answer, go to step 8.

Step 8: If answer doesn't appertain to the answer area, apply constraint handling procedure, otherwise go to step 9.

Step 9: If stopping condition is provided, keep final value of answer, otherwise, go to step 3.

4. Real-world case: Golestan dam in Gonbad Kavous county of Iran

This section includes a case study for applying the mathematical model, and computational results are reported to verify the applicability of the presented approach. A realistic case study of the Golestan dam is introduced in Gonbad Kavous County of Iran to demonstrate the usefulness of the proposed approach. The Golestan dam is built in the city of Gonbad Kavous, about 12 km northeast of the city of Gonbad Kavous in the Gorganrood River. The Golestan dam is one of the types of embankment dams, and its main branch originates in the highlands of the eastern mountains of the province. Figures 1 and 2 show the location of the dam before and after the construction of Golestan Dam. Golestan Province is one of the most populated areas in Iran. The efforts to control the flood in the province are the construction of garden dams on the Gorganrood River. The development aims is to develop the right bank lands by using the irrigation system under pressure at the 10000 hectares of land, helping to improve the existing land area of the Voshmgir dam, equivalent to 9200 ha and control of the annual destructive floods. The dual-purpose use of the dam reservoir for fish farming was done experimentally for several consecutive years while using water for agriculture. The initial design and adjustment capacity, as well as the water permeability of the irrigation canals of the Golestan Dam network for 60% of cereals and 40% of summer crops (corn). Irrigation of about 2050 hectares of land between Golestan Dam and Voshmgir Dam based on the cotton cultivation is another goal of the dam. Golestan dam

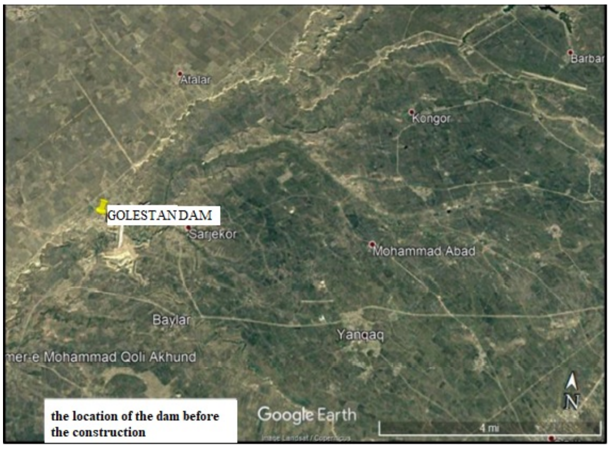

Figure 1: The location of the dam before the construction.

Figure 2: The location of the dam after the construction.

has been put into the operation in 2002. The height of the Golestan dam is 25 meters and the height of the river floor is 17 meters, the length of the dam crown is 1367 meters, the thickness of the dam crown is 200 meters and the width of the crown is 10 meters and the volume of the reservoir is 86 million cubic meters at a normal level. Because the risk is equal to the probability of a multiplication accident in its consequences, in most dams, the risk of 50 to 200-year floods downstream is more or much higher than the risk of floods with a return period of 10,000 years to PMF. Water intake of Golestan dam reservoir is from Gorganrood, Oghan river, Doogh and Aji Su. Figure 3 shows the Golestan dam overflow. Due to the incoming sediments, the volume of sediments in this dam has increased from 86 million cubic meters to 48.8 million cubic meters in 2018. Thus, an average of 2.06 cubic meters of sediment entered the dam annually. Golestan dam bed has deposited sediment with higher height as it approaches the main body, and in some cases, erosion and erosion have occurred on the sides, and the middle line of the reservoir has more sediment height. Golestan dam spillway chute concrete steps of freedom and the ability to pass along 130 meters maximum flow 1550 cubic meters per second. The overflow is located at the mouth of the Ogan River to the reservoir, which is prone to the sediment. According to the history of flooding into the reservoir, likely to cause the congestion and disruption to output dredging water entering the spillway overflow in the event of there. A rational analysis of

Figure 3: Golestan Dam overflow.

the new Chebyshev collocation method and TLBO is reported in the remainder of this section. The proposed Chebyshev collocation approach is coded in MATLAB

(R2018b) for converting OCP to an NLP and then NLP is solved using TLBO in MATLAB on a 64-bit computer, Intel Core i7, 3.3 GHz processor and 4 GB of RAM. A Chebyshev collocation approach is utilized with TLBO to evaluate the solution to the optimality mathematical model of OCP. The following parameters are considered for the TLBO: the population size is 50 and the number of iterations equal to 100 for the case study.

The water is taken through the hydro-suction pipe; the evaporation losses and the dam reservoir's filling function are plotted over 40 years in Figures 4 to 6. In the Golestan dam, due to the lack of water dredging in the hydro-suction pipelines, there is no water, so the amount of water in these pipes is zero. Figure 4 shows the water taken through hydrosuction pipes over a period of 40 years. At the beginning of the dam intake, evaporation losses are increasing since the water level is high and sedimentation has not been done, evaporation losses are increasing. But over time, with increasing sediment in the dam reservoir and decreasing water height, the percentage of evaporation losses decreases. Figure 5 shows the evaporation losses over a period of 40 years. At the beginning of the dam intake, when there is no sediment in the dam reservoir, the filling of the dam reservoir has an increasing trend. In some cases, factors such as the reduced rainfall, drought and the presence of another dam upstream of the reservoir caused a decreasing trend in filling the dam reservoir. Over time, as the cold tank fills with sediment, the volume of water percentage also decreases. Figure 6 shows the filling function of the dam reservoir over a period of 40 years.

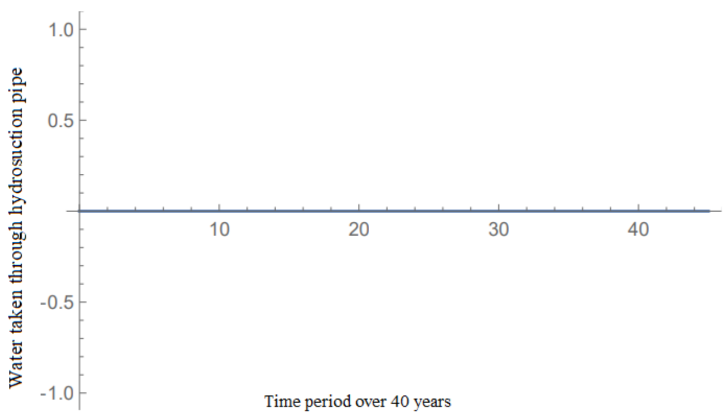

Figure 4: Water taken through hydro-suction pipe.

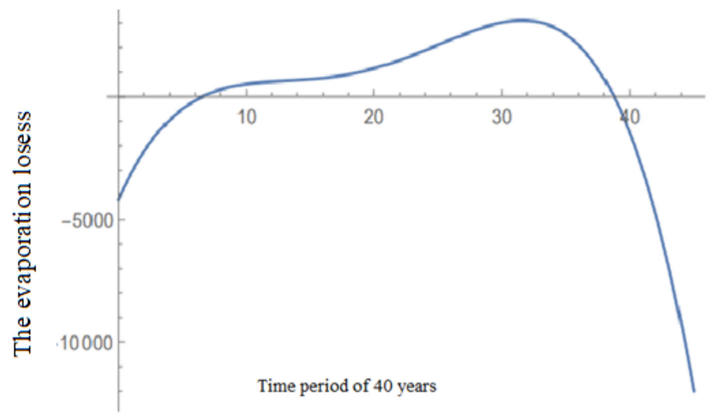

Figure 5: The evaporation losses.

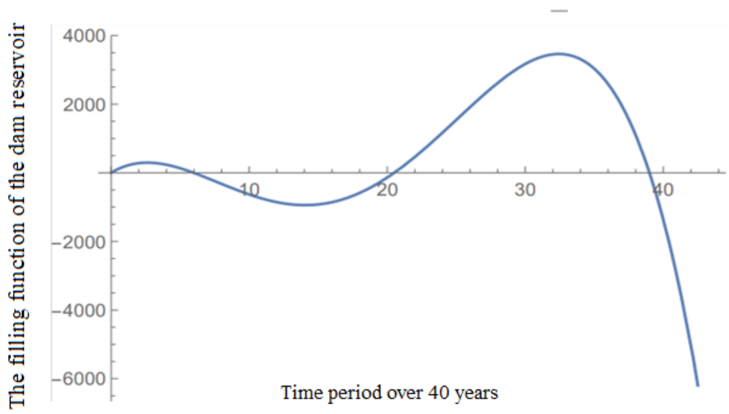

Figure 6: The filling function of the dam reservoir.

5. Conclusions

In this study, a collocation approach based on Lagrange polynomials and TLBO has been developed for solving the OCP of the reservoir sedimentation management on Golestan dam in Gonbad Kavous city, Iran. Finally, based on the numerical results section, optimal decisions can be made for sediment management in Golestan dam. This is the main advantage of this research using the simulation by the OCP. According to the reports received by GRWO, dredging does not take place in Golestan dam, and considering that filling the dam with the sediments reduces the water storage, increases the dam maintenance costs and also reduces the stability of the dam. The results show that in time intervals, the higher the sedimentation rate, the less the dam and the loss increase. In Golestan province, instead of dredging the dam, after filling the dam reservoir, an alternative reservoir will be built and the water in the dam will be directed to the new reservoir, which will increase the cost if the less cost by dredging will be had.

The OCP of sedimentation is a new research field, and due to its widespread use in the field of natural resources and watershed biological operations, with special attention to water and soil, several developments can be considered for it. In the following, we discuss suggestions for future research.

- It seems interesting to consider the issue in a case study in Iran that dredging is suggested as a control operation in reducing sediment from the reservoir;
- Performance evaluation of spectral and Galerkin methods in converting the OCP into an optimization problem and comparing with current results;
- In future studies, we can use other models of metaheuristic algorithms to solve the optimization problem in order to improve objective function values and compare the results with current results;
- Studying the OCP of sedimentation by considering uncertainty in some mathematical model parameters.

Conflicts of Interest. The authors declare that they have no conflicts of interest regarding the publication of this article.

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