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Toplogical and Geometric KM-Single Valued Neutrosophic Metric Spaces

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Abstract

This paper introduces the novel concept of KM-single valued neutrosophic metric spaces as an especial generalization of KM-fuzzy metric spaces, investigates several topological and structural properties and presents some of its applications. This study also considers the metric spaces and constructs KM-single valued neutrosophic spaces with respect to any given triangular norms and triangular conorms. Moreover, we try to extend the concept of KM-single valued neutrosophic metric spaces to a larger class of KM-single valued neutrosophic metric spaces such as union of KM-single valued neutrosophic metric spaces and product of KM-single valued neutrosophic metric spaces.

Keywords:KM-single valued neutrosophic metric, left-continuous triangular (co)norm, Cauchy sequence

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1. Introduction

Classical set theory is a pure concept and without quality or criteria, so it is not attractive to use in our world, that's why we use the neutrosophic sets theory as one of a generalizations of set theory in order to deal with uncertainties, which is a key action in the contemporary world introduced by Smarandache for the first time in

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1998 and 2005 [11]. This concept is a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. This theory describes an important role in modeling and controlling unsure hypersystems in nature, society and industry. In addition, fuzzy topological spaces as a generalization of topological spaces, have a fundamental role in construction of fuzzy metric spaces as an extension of the concept of metric spaces. The theory of fuzzy metric spaces works on finding the distance between two points as non-negative fuzzy numbers, which have various applications. The structure of fuzzy metric spaces is equipped with mathematical tools such as triangular norms and fuzzy subsets depending on time parameter and on other variables. This theory has been proposed by different researchers with different definitions from several points of views ([1, 2, 3, 7]), and that this study was applied to the notion of KM-fuzzy metric space introduced in 1975 [2] by Kramosil and Michalek. Further materials regarding the single valued neutrosophic metric sets and their applications in, graphs, hypergraphs and neutro algebras are available in the literature too [4, 5, 6].

Regarding these points, we introduce the concept of KM-single valued neutrosophic metric spaces as an application of neutrosophic sets. Although we apply three fuzzy subsets in our definition but it has limited their sum of three fuzzy subsets in to a fuzzy subset. Also we proved that KM-single valued neutrosophic metric spaces have both non-increasing fuzzy subsets and non-decreasing fuzzy subsets. It has tried to construct a larger class of KM-single valued neutrosophic metric spaces with respect to union and product operations. Metric spaces have an important role in generating the KM-single valued neutrosophic metric spaces via any triangular norms and triangular conorms; therefore, we analyzed the relation between the class of metric spaces and KM-single valued neutrosophic metric spaces. Moreover, we presented the ball subsets in KM-single valued neutrosophic metric spaces and proved that ball subsets are open subsets. Furthermore, the present study aimed to generate some topologies on the base set of KM-single valued neutrosophic metric spaces with respect to open balls and it is one of the main motivations of introducing the KM-single valued neutrosophic metric spaces. The KM-single valued neutrosophic metric spaces are not necessarily infinite spaces. Thus, another important motivation of this study is the construction of finite KM-single valued neutrosophic metric spaces. This study also presented an induced equivalence in relation to KM-single valued neutrosophic metric spaces such that a quotient of given KM-single valued neutrosophic metric space is a KM-single valued neutrosophic metric space. This study generated some metrics on any nonempty sets using the concept of KM-single valued neutrosophic metric spaces and extended KM-single valued neutrosophic metric spaces to a family of metric spaces with left-continuous metrics. For the significance of the applicability of this argument, we presented an example of application of KM-single valued neutrosophic metric space on economic and it encouraged us to develop this study.

2. Preliminaries

This section presented some definitions and results which are used in following sections.

Definition 2.1. [11] Let V be a universal set. A neutrosophic set (NS) X in V is an object has the following form $X = \{(x, T_X(x), I_X(x), F_X(x)) \mid x \in V\}$, or $X : V \to [0,1]\times[0,1]\times[0,1]$ which is characterized by a truth-membership function T_X, an indeterminacy-membership function I_X and a falsity-membership function F_X. There is no restriction on the sum of $T_X(x), I_X(x)$ and $F_X(x)$, therefore $0^- \leq \sup T_X(x) + \sup I_X(x) + \sup F_X(x) \leq 3^+$.

Definition 2.2. [10] A binary operation $T : [0,1] \times [0,1] \to [0,1]$ is a t-norm if it for all $x, y, z, w \in [0,1]$ satisfies the following:

(i) $T(1, x) = x$;

(ii) $T(x, y) = T(y, x)$;

(iii) $T(T(x, y), z) = T(x, T(y, z))$;

(iii) If $w \leq x$ and $y \leq z$ then $T(w, y) \leq T(x, z)$.

Definition 2.3. [8] A triplet (X, ρ, T) is called a KM-fuzzy metric space, if X is an arbitrary non–empty set, T is a left-continuous t-norm and $\rho : X^2 \times \mathbb{R}^{\geq 0} \to [0,1]$ is a fuzzy set, such that for each $x, y, z, \in X$ and $t, s \geq 0$, we have:

(i) $\rho(x, y, 0) = 0$,

(ii) $\rho(x, x, t) = 1$ for all $t > 0$,

(iii) $\rho(x, y, t) = \rho(y, x, t)$(commutative property),

(iv) $T(\rho(x, y, t), \rho(y, z, s)) \leq \rho(x, z, t + s)$(triangular inequality),

(vi) $\rho(x, y, -) : \mathbb{R}^{\geq 0} \to [0,1]$ is a left-continuous map,

(vii) $\rho(x, y, t) \to 1$, when $t \to \infty$.

$(viii)$ $\rho(x, y, t) = 1, \forall\ t > 0$ implies that $x = y$.

If (X, ρ, T) is satisfied in conditions (i)–(vii), then it is called a KM-fuzzy pseudometric space and ρ is called a KM-fuzzy pseudometric.

3. KM-Single Valued Neutrosophic Metric Space

In this section, we introduce the concept of KM-single valued neutrosophic metric spaces and investigate their properties. In addition, we generate KM-single valued neutrosophic metric spaces with respect to metric spaces.

Definition 3.1. A triplet $(X, \rho_1, \rho_2, \rho_3, T, S)$ is called a KM-single valued neutrosophic metric space, if X is an arbitrary nonempty set, T is a left-continuous t-norm, S is a left-continuous t-conorm, and $\rho_1, \rho_2, \rho_3 : X^2 \times \mathbb{R}^{\geq 0} \to [0,1]$ are fuzzy subsets, such that for each $x, y, z, \in X$ and $t, s \geq 0$, we have:

(i) $\rho_1(x, y, 0) = 0$ and for all $i \in \{2,3\}$, $\rho_i(x, y, 0) = 1$,

(ii) $\rho_1(x, x, t) = 1$ and for all $i \in \{2,3\}$, $\rho_i(x, x, t) = 0$, where $t > 0$,

(iii) for all $i \in \{1,2,3\}$, $\rho_i(x, y, t) = \rho_i(y, x, t)$(commutative property),

(iv) $T(\rho_1(x, y, t), \rho_1(y, z, s)) \leq \rho_1(x, z, t+s)$ and for all $i \in \{2,3\}, S(\rho_i(x, y, t), \rho_i(y, z, s)) \geq \rho_i(x, z, t+s)$ (triangular inequality),

(vi) for all $i \in \{1,2,3\}$, $\rho_i(x, y, -) : \mathbb{R}^{\geq 0} \to [0,1]$ are left-continuous maps,

(vii) $\lim_{t\to\infty} \rho_1((x, y, t)) = 1$ and for all $i \in \{2,3\}$, $\lim_{t\to\infty} \rho_i((x, y, t)) = 0$,

$(viii)$ for all $t \in \mathbb{R}^+$ and for all $x, y \in X$, we have $0 \leq \sum_{i=1}^{3} \rho_i(x, y, t) \leq 1$,

(ix) $\forall\ t > 0,\ \rho_1(x, y, t) = 1$ implies that $x = y$ and for all $i \in \{2,3\}$, $\forall\ t > 0,\ \rho_i(x, y, t) = 0$ implies that $x = y$.

If $(X, \rho_1, \rho_2, \rho_3, T, S)$ satisfies in conditions (i)–$(viii)$, then it is called a KM-single valued neutrosophic pseudometric space and triple (ρ_1, ρ_2, ρ_3) is called a KM-single valued neutrosophic pseudometric.

The following proposition shows that KM-single valued neutrosophic metrics are different from KM-fuzzy metrics.

Proposition 3.2. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space. Then for all $x, y \in X$ and $t \in \mathbb{R}^+$*

(i) $\rho_1(x, y, t) + \rho_2(x, y, t) + \rho_3(x, y, t) \neq 0$;

(ii) $\rho_2(x, y, t) + \rho_3(x, y, t) \neq 1$;

(iii) *if* $\rho_2(x, y, t) = \rho_3(x, y, t)$, *then* $\rho_2(x, y, t) < \frac{1}{2}$, *where* $t > 0$;

(iv) $\rho_1(x, y, t) = 1$, *if and only if* $\rho_2(x, y, t) + \rho_3(x, y, t) = 0$.

Proof. It is immediate by definition. □

From now on, for all $x, y \in [0,1]$ we consider $T_{min}(x,y) = \min\{x,y\}$, $T_{pr}(x,y) = xy$, $T_{lu}(x,y) = \max(0, x+y-1)$, $T_{do}(x,y) = \begin{cases} \dfrac{xy}{x+y-xy} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$,

$\mathcal{C}_T = \{T : [0,1] \times [0,1] \to [0,1] \mid T \text{ is a left-continuous t-norm}\}$, $S_{max}(x,y) = \max\{x,y\}$, $S_{pr}(x,y) = x+y-xy$, $S_{lu}(x,y) = \min(1, x+y)$ and $\mathcal{C}_S = \{S : [0,1] \times [0,1] \to [0,1] \mid S \text{ is a left-continuous t-conorm}\}$.

In what follows, we investigate some properties of the KM-single valued neutrosophic metric spaces.

Definition 3.3. Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space,let $(x_n)_n$ be a sequence in X and $x \in X$. We say that

(*i*) $(x_n)_n$ converges to x, if for all $t \in \mathbb{R}^+$ we have $\lim\limits_{n\to\infty} \rho_1(x_n, x, t) = 1$ and $\lim\limits_{n\to\infty} \rho_2(x_n, x, t) = \lim\limits_{n\to\infty} \rho_3(x_n, x, t) = 0$. It means that for all $t \in \mathbb{R}^+$ and for all $0 < \epsilon < 1$, there exists $N \in \mathbb{N}$, such that for all $n \geq N$, $1 - \epsilon < \rho_1(x_n, x, t), \rho_2(x_n, x, t) < \epsilon$ and $\rho_3(x_n, x, t) < \epsilon$.

(*ii*) $(x_n)_n$ is a Cauchy sequence if and only if for each $t \in \mathbb{R}^+$ and $p > 0$ $\lim\limits_{n\to\infty} \rho_1(x_n, x_{n+p}, t) = 1$ and $\lim\limits_{n\to\infty} \rho_2(x_n, x_{n+p}, t) = \lim\limits_{n\to\infty} \rho_3(x_n, x_{n+p}, t) = 0$

In [3], George and Veeramani proved that every GV-fuzzy metric is a non-decreasing map. In a similar way we have the following Theorem on the KM-single valued neutrosophic metric spaces.

Theorem 3.4. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space. Then $\rho_1(x, y, -) : \mathbb{R}^{\geq 0} \to [0,1]$ is a non-decreasing map and for all $i \in \{2,3\}$, $\rho_i(x, y, -) : \mathbb{R}^{\geq 0} \to [0,1]$ are non-increasing maps.*

Proof. Let $0 \leq t < s$. If $t = 0$, then for all $t > 0, \rho_2(x, y, 0) = 1 \geq \rho_2(x, y, s)$. But for all $t \neq s$ if $\rho_2(x, y, t) < \rho_2(x, y, s)$, then $S(\rho_2(x, y, t), \rho_2(y, y, s-t)) \geq \rho_2(x, y, s)$. By definition, $\rho_2(y, y, s-t) = 0$, and thus obtain that $\rho_2(x, y, t) \geq \rho_2(x, y, s)$, which is a contradiction and it implies that $\rho_2(x, y, -) : \mathbb{R}^{\geq 0} \to [0,1]$ is a non-increasing map. In a similar way, $\rho_3(x, y, -) : \mathbb{R}^{\geq 0} \to [0,1]$ is a non-increasing map and $\rho_1(x, y, -) : \mathbb{R}^{\geq 0} \to [0,1]$ is a non-decreasing map. □

In [12], Rodrguez-Lopez and Romaguera, proved that every GV-fuzzy metric is a continuous map. In a similar way, one can see that KM-single valued neutrosophic metrics are left-continuous maps.

Theorem 3.5. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space. Then for all $i \in \{1,2,3\}$, ρ_i's are left-continuous maps on $X^2 \times \mathbb{R}^{\geq 0}$.*

Proof. Let $x, y \in X, t \in \mathbb{R}^+$ and $(x'_n, y'_n, t'_n)_n$ be a sequence in $X^2 \times \mathbb{R}^{\geq 0}$ that $\lim\limits_{n\to\infty} (x'_n, y'_n, t'_n) = (x, y, t)$. Suppose that $t/2 > \epsilon > 0$ be arbitrary. Since $(\rho_1(x'_n, y'_n, t'_n))_n$ is a sequence in $[0,1]$, there is a subsequence $(x_n, y_n, t_n)_n$ of

$(x'_n, y'_n, t'_n)_n$ such that $\rho_2((x_n, y_n, t_n))$ converges to some points of $[0, 1]$. Now, $\lim_{n\to\infty} t_n = t$ implies that there is $N \in \mathbb{N}$ such that for all $n \geq N$ we have $t - \epsilon < t_n$. Hence, for all $n \geq N$ we have $\rho_2(x_n, y_n, t_n) \leq \rho_2(x_n, y_n, t - \epsilon) \leq S(\rho_2(x_n, x, \epsilon/2), \rho_2(x, y, t-2\epsilon), \rho_2(y, y_n, \epsilon/2))$ and $\rho_2(x, y, t+2\epsilon) \leq \rho_2(x, y, t_n+\epsilon) \leq S(\rho_2(x_n, x, \epsilon/2), \rho_2(x_n, y_n, t_n), \rho_2(y, y_n, \epsilon/2))$. Thus $\lim_{n\to\infty} \rho_2(x_n, y_n, t_n) \leq S(0, \rho_2(x, y, t - 2\epsilon), 0) = \rho_2(x, y, t - 2\epsilon)$, so by left-continuity of the function ρ_2 on $\mathbb{R}^{\geq 0}$, get that $\lim_{\epsilon\to 0}(\lim_{n\to\infty} \rho_2(x_n, y_n, t_n)) \leq \lim_{\epsilon\to 0}(\rho_2(x, y, t - 2\epsilon)) = \rho_2(x, y, t)$. In addition, $\rho_2(x, y, t + 2\epsilon) \geq S(0, \lim_{n\to\infty} \rho_2(x_n, y_n, t_n), 0) = \lim_{n\to\infty} \rho_2(x_n, y_n, t_n)$. Thus $\rho_2(x, y, t + 2\epsilon) \leq \lim_{n\to\infty} \rho_2(x_n, y_n, t_n)$ and by left-continuity of the function ρ_2 on $\mathbb{R}^{\geq 0}$, get that $\lim_{\epsilon\to 0}(\lim_{n\to\infty} \rho_2(x_n, y_n, t_n)) \geq \lim_{\epsilon\to 0}(\rho_2(x, y, t + 2\epsilon)) = \rho_2(x, y, t)$. It follows that ρ_2 is a left-continuous map on $X^2 \times \mathbb{R}^{\geq 0}$. In a similar way one can see that ρ_1 and ρ_3 are left-continuous maps on $X^2 \times \mathbb{R}^{\geq 0}$. □

Let $x_1, x_2, \ldots, x_n \in [0, 1]$. Then for all $T \in \mathcal{C}_T$ and $S \in \mathcal{C}_S$, we have $T(x_1, x_2, x_3) = T(T(x_1, x_2), x_3)$ and $T(x_1, x_2, \ldots, x_{n-1}, x_n) = T(T(x_1, x_2, \ldots, x_{n-1}), x_n)$. In a similar way $S(x_1, x_2, x_3) = S(S(x_1, x_2), x_3)$ and $S(x_1, x_2, \ldots, x_{n-1}, x_n) = S(S(x_1, x_2, \ldots, x_{n-1}), x_n)$.

Lemma 3.6. *Let $x_1, x_2, \ldots, x_n \in [0, 1]$, $T \in \mathcal{C}_T$ and $S \in \mathcal{C}_S$. Then*

(i) *if $0 \in \{x_1, x_2, \ldots, x_n\}$, then $T(x_1, x_2, \ldots, x_{n-1}, x_n) = 0$;*

(ii) *if $1 \in \{x_1, x_2, \ldots, x_n\}$, then $S(x_1, x_2, \ldots, x_{n-1}, x_n) = 1$;*

(iii) *$S_{max}(x_1, x_2, \ldots, x_{n-1}, x_n) \leq S(x_1, x_2, \ldots, x_{n-1}, x_n)$;*

(iv) *$T(x_1, x_2, \ldots, x_{n-1}, x_n) \leq T_{min}(x_1, x_2, \ldots, x_{n-1}, x_n)$;*

(v) *$T(x_1, x_2, \ldots, x_{n-1}, x_n) = 1$ if and only if $x_1 = x_2 = \ldots = x_{n-1} = x_n = 1$;*

(vi) *$S(x_1, x_2, \ldots, x_{n-1}, x_n) = 0$ if and only if $x_1 = x_2 = \ldots = x_{n-1} = x_n = 0$;*

(vii) *for all $p \in \mathbb{R}^+$, $S_{max}(\frac{x_1}{p}, \frac{x_2}{p}, \ldots, \frac{x_{n-1}}{p}, \frac{x_n}{p}) = \frac{1}{p} S_{max}(x_1, x_2, \ldots, x_{n-1}, x_n)$ and $T_{min}(\frac{x_1}{p}, \frac{x_2}{p}, \ldots, \frac{x_{n-1}}{p}, \frac{x_n}{p}) = \frac{1}{p} T_{min}(x_1, x_2, \ldots, x_{n-1}, x_n)$.*

Theorem 3.7. *If $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ is a KM-fuzzy metric space, $T \in \mathcal{C}_T$ and $S \in \mathcal{C}_S$. Then $(X, \rho_1, \rho_2, \rho_3, T, S)$ is a KM-single valued neutrosophic metric space.*

Proof. It is clear. □

In the following, we generate the KM-single valued neutrosophic metric spaces with respect to the metric spaces.

For all $x, y \in X$ and for all $0 > p, p', m, (p, p' \geq 3m), t, s \in \mathbb{R}^{\geq 0}$, define $\rho_1(x,y,t) = \begin{cases} 0 & t = 0 \\ \dfrac{\varphi(t)}{\varphi(t) + md(x,y)} & t \neq 0 \end{cases}$, $\rho_2(x,y,t) = \begin{cases} \dfrac{md(x,y)}{p(md(x,y) + \varphi(t))} & \text{if } t > 0 \\ 1 & \text{if } t = 0 \end{cases}$ and $\rho_3(x,y,t) = \begin{cases} \dfrac{md(x,y)}{p'(md(x,y) + \varphi(t))} & \text{if } t > 0 \\ 1 & \text{if } t = 0 \end{cases}$, where $\varphi : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0}$ is increasing the left-continuous map and $\varphi(t) + md(x,y) \neq 0$ and $\varphi(t+s) \geq \varphi(t) + \varphi(s)$.

Theorem 3.8. *Let (X, d) be a metric space. If $p', p > 2$, then $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ is a KM-single valued neutrosophic metric space.*

Proof. If $p, p' > 2$, then for all $x, y \in X, t \in \mathbb{R}^+$, we have $0 \leq \sum_{i=1}^{3} \rho_i(x,y,t) \leq 1$. Now, we only prove the triangular inequality property. Let $x, y, z \in X$. For $0 \in \{t, s\}$ is clear, now for $0 \notin \{t, s\}$ we investigate it. Without loss of generality, if $\dfrac{\varphi(t)}{p(\varphi(t) + md(x,y))} \leq \dfrac{\varphi(s)}{p(\varphi(s) + md(z,y))}$, we get that $\varphi(t)d(z,y) \leq \varphi(s)d(x,y)$. Since for all $s, t, m \in \mathbb{R}^+, \varphi(t+s) \geq \varphi(t) + \varphi(s)$, we get that $\varphi(t)d(x,z) \leq \varphi(t+s)d(x,y)$ and so

$$T_{min}(\frac{\varphi(t)}{p(\varphi(t) + md(x,y))}, \frac{\varphi(s)}{p(\varphi(s) + md(y,z))}) \leq \frac{\varphi(t+s)}{p(\varphi(t+s) + md(x,z))}.$$

In a similar way, one can see that $S_{max}(\dfrac{md(x,y)}{p(md(x,y) + \varphi(t))}, \dfrac{md(z,y)}{p(md(z,y) + \varphi(s))}) \geq \dfrac{md(x,z)}{p(md(x,z) + \varphi(t+s))}$, where $i \in \{2, 3\}$. It follows that $T_{min}(\rho(x,y,t), \rho_1(y,z,s)) \leq \rho_1(x,z,t{+}s), S_{max}(\rho_i(x,y,t), \rho(y,z,s)) \geq \rho(x,z,t{+}s)$ and so $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ is a KM-single valued neutrosophic metric space. □

By Lemma 3.6 and Theorem 3.8, one can construct a KM-single valued neutrosophic metric space with any triangular norms and triangular conorms on any given metric space as follows.

Corollary 3.9. *Let (X, d) be a metric space. Then there exist fuzzy subsets ρ_1, ρ_2, ρ_3 on $X^2 \times \mathbb{R}^{\geq 0}$ such that for each $T \in \mathcal{C}_T$ and $S \in \mathcal{C}_S$, $(X, \rho_1, \rho_2, \rho_3, T, S)$ is a KM-single valued neutrosophic metric space.*

Example 3.10. Consider $X = \mathbb{N}$ and the metric space (X, d), where $d(x,y) = |x - y|$.

Define for all $x, y \in X$, $\rho_1(x,y,t) = \begin{cases} \dfrac{3^{8t}}{(3^{8t} + 7d(x,y))} & \text{if } t > 0 \\ 0 & \text{if } t = 0 \end{cases}$, $\rho_2(x,y,t) = \begin{cases} \dfrac{7d(x,y)}{21(7d(x,y) + 3^{8t})} & \text{if } t > 0 \\ 1 & \text{if } t = 0 \end{cases}$, and $\rho_3(x,y,t) = \begin{cases} \dfrac{7d(x,y)}{37(7d(x,y) + 3^{8t})} & \text{if } t > 0 \\ 1 & \text{if } t = 0 \end{cases}$.

Then $(X, \rho_1, \rho_2, \rho_3, T, S)$ is a KM-single valued neutrosophic metric space, where $T \in \mathcal{C}_T$ and $S \in \mathcal{C}_S$.

3.1. Operations on KM-Single Valued Neutrosophic Metric Spaces

In this subsection, we extend KM-single valued neutrosophic metric spaces to union of KM-single valued neutrosophic metric spaces and product of KM-single valued neutrosophic metric spaces. Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ and $(Y, \rho'_1, \rho'_2, \rho'_3, T, S)$ be KM-single valued neutrosophic metric spaces, $(x_1, y_1), (x_2, y_2) \in X \times Y$ and $t \in \mathbb{R}^{\geq 0}$. For an arbitrary $T \in \mathcal{C}_T$ and $S \in \mathcal{C}_S$ define $T(\rho_1, \rho'_1), S(\rho_2, \rho'_2), S(\rho_3, \rho'_3) : (X \times Y)^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$ by $T(\rho_1, \rho'_1)\big((x_1, y_1), (x_2, y_2), t\big) = T(\rho_1(x_1, x_2, t), \rho'_1(y_1, y_2, t))$, $S(\rho_2, \rho'_2)\big((x_1, y_1), (x_2, y_2), t\big) = S(\rho_2(x_1, x_2, t), \rho'_2(y_1, y_2, t))$ and

$$S(\rho_3, \rho'_3)\big((x_1, y_1), (x_2, y_2), t\big) = S(\rho_3(x_1, x_2, t), \rho'_3(y_1, y_2, t)\big).$$

So we have the following theorem.

Theorem 3.1. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ and $(Y, \rho'_1, \rho'_2, \rho'_3, T, S)$ be KM-single valued neutrosophic metric spaces. Then $(X \times Y, T_{min}(\rho_1, \rho'_1), S_{max}(\rho_2, \rho'_2), S_{max}(\rho_3, \rho'_3), T, S)$ is a KM-single valued neutrosophic metric space.*

Proof. Let $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in X \times Y$ and $t, s \in \mathbb{R}^{\geq 0}$.

(i) Since for all $x_1, x_2 \in X, y_1, y_2 \in Y, \rho_1(x_1, x_2, 0) = 0$ and $\rho_2(x_1, x_2, 0) = \rho_3(x_1, x_2, 0) = \rho'_2(y_1, y_2, 0) = \rho'_3(y_1, y_2, 0) = 1$, we have $T_{min}(\rho_1, \rho'_1)\big((x_1, y_1), (x_2, y_2), 0\big) = 0$, $S_{max}(\rho_2, \rho'_2)\big((x_1, y_1), (x_2, y_2), 0\big) = 1$ and $S_{max}(\rho_3, \rho'_3)\big((x_1, y_1), (x_2, y_2), 0\big) = 1$.

(ii) $T_{min}(\rho_1, \rho'_1)\big((x_1, y_1), (x_2, y_2), t\big) = 1$ if and only if $\rho_1(x_1, x_2, t) = \rho'_1(y_1, y_2, t) = 1$ if and only if $(x_1, y_1) = (x_2, y_2)$. In addition, $S_{max}(\rho_2, \rho'_2)\big((x_1, y_1), (x_2, y_2), t\big) = 0$ if and only if $\rho_2(x_1, x_2, t) = \rho'_2(y_1, y_2, t) = 0$ if and only if $(x_1, y_1) = (x_2, y_2)$. In a similar way, $S_{max}(\rho_3, \rho'_3)\big((x_1, y_1), (x_2, y_2), t\big) = 0$ if and only if $(x_1, y_1) = (x_2, y_2)$.

(iii) It is clear that $T_{min}(\rho_1, \rho'_1), S_{max}(\rho_2, \rho'_2)$ and $S_{max}(\rho_3, \rho'_3)$ are commutative maps.

(iv) By Lemma 3.6,

$$\begin{aligned}
&T\big(T_{min}(\rho_1, \rho'_1)((x_1, y_1), (x_2, y_2), t), T_{min}(\rho_1, \rho'_1)((x_2, y_2), (x_3, y_3), s)\big)\\
&= T\big(T_{min}(\rho_1(x_1, x_2, t), \rho'_1(y_1, y_2, t)), T_{min}(\rho_1(x_2, x_3, s), \rho'_1(y_2, y_3, s))\big)\\
&\leq T_{min}(T\big(\rho_1(x_1, x_2, t), \rho_1(x_2, x_3, s)\big), T\big(\rho'_1(y_1, y_2, t), \rho'_1(y_2, y_3, s)\big)))\\
&\leq T_{min}(\rho_1(x_1, x_3, t + s), \rho'_1(y_1, y_3, t + s))\\
&= T_{min}(\rho_1, \rho'_1)((x_1, y_1), (x_3, y_3), t + s).
\end{aligned}$$

Also,

$$
\begin{aligned}
&S\big(S_{max}(\rho_2,\rho_2')((x_1,y_1),(x_2,y_2),t), S_{max}(\rho_2,\rho_2')((x_2,y_2),(x_3,y_3),s)\big)\\
&= S\big(S_{max}(\rho_2(x_1,x_2,t),\rho_2'(y_1,y_2,t)), S_{max}(\rho_2(x_2,x_3,s),\rho_2'(y_2,y_3,s))\big)\\
&\geq S_{max}(S\big(\rho_2(x_1,x_2,t),\rho_2(x_2,x_3,s)\big), S\big(\rho_2'(y_1,y_2,t),\rho_2'(y_2,y_3,s)\big)))\\
&\geq S_{max}(\rho_2(x_1,x_3,t+s),\rho_2'(y_1,y_3,t+s))\\
&= S_{max}(\rho_2,\rho_2')((x_1,y_1),(x_3,y_3),t+s).
\end{aligned}
$$

In a similar way, one can see that

$$
\begin{aligned}
&S\big(S_{max}(\rho_3,\rho_3')((x_1,y_1),(x_2,y_2),t), S_{max}(\rho_3,\rho_3')((x_2,y_2),(x_3,y_3),s)\big)\\
&\geq S_{max}(\rho_3,\rho_3')((x_1,y_1),(x_3,y_3),t+s).
\end{aligned}
$$

(v) Since $\rho_1,\rho_2,\rho_3,\rho_1',\rho_2',\rho_3'$ are left-continuous maps, we get that $T_{min}(\rho_1,\rho_1'), S_{max}(\rho_2,\rho_2'), S_{max}(\rho_3,\rho_3')$ are left-continuous map.
(vi) Since T_{min} and S_{max} are left-continuous maps, we get that

$$
\begin{aligned}
&\lim_{t\to\infty} T_{min}(\rho_1(x_1,x_2,t),\rho_1'(y_1,y_2,t))\\
&= T_{min}(\lim_{t\to\infty}\rho_1(x_1,x_2,t), \lim_{t\to\infty}\rho_1'(y_1,y_2,t)) = T_{min}(1,1) = 1.
\end{aligned}
$$

In a similar way,

$$\lim_{t\to\infty} S_{max}(\rho_2(x_1,x_2,t),\rho_2'(y_1,y_2,t)) = 1,$$

and

$$\lim_{t\to\infty} S_{max}(\rho_3(x_1,x_2,t),\rho_3'(y_1,y_2,t)) = 1.$$

The other cases, clearly obtained, so

$$(X\times Y, T_{min}(\rho_1,\rho_1'), S_{max}(\rho_2,\rho_2'), S_{max}(\rho_3,\rho_3'), T, S)$$

is a KM-single valued neutrosophic metric space. □

Let $X\cap Y=\emptyset$, $(X,\rho_1,\rho_2,\rho_3,T,S)$ and $(Y,\rho_1',\rho_2',\rho_3',T,S)$ be KM-single valued neutrosophic metric spaces, $x,y\in X\cup Y$ and $t\in\mathbb{R}^{\geq 0}$. Consider $\epsilon(x,y,t) = \bigwedge_{\substack{x,u\in X\\ y,v\in Y}} (\rho_1'(x,u,t)\wedge\rho_1'(y,v,t)))$, $\sigma(x,y,t) = \bigvee_{\substack{x,u\in X\\ y,v\in Y}} (\rho_2(x,u,t)\vee\rho_2'(y,v,t)))$ and $\delta(x,y,t) = \bigvee_{\substack{x,u\in X\\ y,v\in Y}} (\rho_3(x,u,t)\vee\rho_3'(y,v,t)))$. Define $\rho_1\cup\rho_1', \rho_2\cup\rho_2', \rho_3\cup\rho_3' : (X\cup Y)^2\times\mathbb{R}^{\geq 0}\to$

$[0,1]$ by

$$(\rho_1 \cup \rho'_1)(x,y,t) = \begin{cases} \rho_1(x,y,t) & \text{if } x,y \in X, \\ \rho'_1(x,y,t) & \text{if } x,y \in Y, \\ \epsilon(x,y,t) & \text{if } x \in X, y \in Y, \end{cases},$$

$$(\rho_2 \cup \rho'_2)(x,y,t) = \begin{cases} \rho_2(x,y,t) & \text{if } x,y \in X, \\ \rho'_2(x,y,t) & \text{if } x,y \in Y, \\ \sigma(x,y,t) & \text{if } x \in X, y \in Y, \end{cases}$$

and $(\rho_3 \cup \rho'_3)(x,y,t) = \begin{cases} \rho_3(x,y,t) & \text{if } x,y \in X, \\ \rho'_3(x,y,t) & \text{if } x,y \in Y, \\ \delta(x,y,t) & \text{if } x \in X, y \in Y, \end{cases}$. So we have the following theorem.

Theorem 3.2. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ and $(Y, \rho'_1, \rho'_2, \rho'_3, T, S)$ be KM-single valued neutrosophic metric spaces. Then $(X \cup Y, \rho_1 \cup \rho'_1, \rho_2 \cup \rho'_2, \rho_3 \cup \rho'_3, T, S)$ is a KM-single valued neutrosophic metric space, where $X \cap Y = \emptyset$.*

Proof. Let $x, y, z \in X \cup Y$ and $t, s \in \mathbb{R}^{\geq 0}$. We only prove the triangular inequality property and other cases are immediate. Let $x, y \in X$(for $x, y \in Y$ is similar), then $T\big((\rho_1 \cup \rho'_1)(x,y,t), (\rho_1 \cup \rho'_1)(y,z,s)\big) = T\big(\rho_1(x,y,t), (\rho_1 \cup \rho'_1)(y,z,s)\big)$. If $z \in X$, then $T\big((\rho_1 \cup \rho'_1)(x,y,t), (\rho_1 \cup \rho'_1)(y,z,s)\big) = T\big(\rho_1(x,y,t), \rho_1(y,z,s)\big) \leq \rho_1(x,z,t+s) = (\rho_1 \cup \rho'_1)(x,z,t+s)$. If $z \in Y$, then $T\big((\rho_1 \cup \rho'_1)(x,y,t), (\rho_1 \cup \rho'_1)(y,z,s)\big) = T\big(\rho_1(x,y,t), \epsilon\big) \leq \epsilon = (\rho_1 \cup \rho'_1)(x,z,t+s)$. Let $x \in X, y \in Y$. Then $T\big((\rho_1 \cup \rho'_1)(x,y,t), (\rho_1 \cup \rho'_1)(y,z,s)\big) = T\big(\epsilon, (\rho_1 \cup \rho'_1)(y,z,s)\big)$. If $z \in Y$, since $x \in X$ and $y \in Y$, we get that $(\rho_1 \cup \rho'_1)(x,z,t+s) = \epsilon$ and so $T\big(\epsilon, (\rho_1 \cup \rho'_1)(y,z,s)\big) = T\big(\epsilon, \rho_2(y,z,s)\big) \leq \epsilon = (\rho_1 \cup \rho'_1)(x,z,t+s)$. If $z \in X$, since $x \in X$ and $y \in Y$, we get that $(\rho_1 \cup \rho'_1)(x,z,t+s) \neq \epsilon$ and so $T\big(\epsilon, (\rho_1 \cup \rho'_1)(y,z,s)\big) = T\big(\epsilon, \epsilon\big) \leq \epsilon \leq \rho_1(x,z,t+s)\big) = (\rho_1 \cup \rho'_1)(x,z,t+s)$.

Suppose that $x, y \in X$(for $x, y \in Y$ is similar), then $S\big((\rho_2 \cup \rho'_2)(x,y,t), (\rho_2 \cup \rho'_2)(y,z,s)\big) = S\big(\rho_2(x,y,t), (\rho_2 \cup \rho'_2)(y,z,s)\big)$. If $z \in X$, then $S\big((\rho_2 \cup \rho'_2)(x,y,t), (\rho_2 \cup \rho'_2)(y,z,s)\big) = S\big(\rho_2(x,y,t), \rho_2(y,z,s)\big) \geq \rho_2(x,z,t+s) = (\rho_2 \cup \rho'_2)(x,z,t+s)$. If $z \in Y$, then $S\big((\rho_2 \cup \rho'_2)(x,y,t), (\rho_2 \cup \rho'_2)(y,z,s)\big) = S\big(\rho_2(x,y,t), \sigma\big) \geq \sigma = (\rho_2 \cup \rho'_2)(x,z,t+s)$. Let $x \in X, y \in Y$. Then $S\big((\rho_2 \cup \rho'_2)(x,y,t), (\rho_2 \cup \rho'_2)(y,z,s)\big) = S\big(\sigma, (\rho_2 \cup \rho'_2)(y,z,s)\big)$. If $z \in Y$, since $x \in X$ and $y \in Y$, we get that $(\rho_2 \cup \rho'_2)(x,z,t+s) = \sigma$ and so $S\big(\sigma, (\rho_2 \cup \rho'_2)(y,z,s)\big) = S\big(\sigma, \rho_2(y,z,s)\big) \geq \sigma = (\rho_2 \cup \rho'_2)(x,z,t+s)$. If $z \in X$, since $x \in X$ and $y \in Y$, we get that $(\rho_2 \cup \rho'_2)(x,z,t+s) \neq \sigma$ and so $S\big(\sigma, (\rho_2 \cup \rho'_2)(y,z,s)\big) = S\big(\sigma, \sigma\big) \geq \sigma \geq \rho_2(x,z,t+s)\big) = (\rho_2 \cup \rho'_2)(x,z,t+s)$. In common a way, we can prove that for all $x, y \in X \cup Y$, $S\big((\rho_3 \cup \rho'_3)(x,y,t), (\rho_3 \cup \rho'_3)(y,z,s)\big) \geq (\rho_3 \cup \rho'_3)(x,z,t+s)$. $(X \cup Y, \rho_1 \cup \rho'_1, \rho_2 \cup \rho'_2, \rho_3 \cup \rho'_3, T, S)$ is a KM-single valued neutrosophic metric space, where $X \cap Y = \emptyset$. $\square$

4. Induced Topology from KM-Single Valued Neutrosophic Metric Space

In this section, in KM-single valued neutrosophic metric space $(X, \rho_1, \rho_2, \rho_3, T, S)$, we introduce the subsets as balls and show that they are open subsets. Also we prove that every KM-single valued neutrosophic metrics ρ_1, ρ_2, ρ_3 on X which has as a base the family of open sets of the form $\mathcal{O} = \{O(x, \epsilon, t) \mid x \in X, 0 < \epsilon < 1, t \in \mathbb{R}^+\}$.

Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space, $t \in \mathbb{R}^{\geq 0}, x \in X$ and $0 < \epsilon < 1$. Define $O_{\rho_1}(x, \epsilon, t) = \{y \in X \mid \rho_1(x, y, t) > 1 - \epsilon\}, O_{\rho_2}(x, \epsilon, t) = \{y \in X \mid \rho_2(x, y, t) < \epsilon\}, O_{\rho_3}(x, \epsilon, t) = \{y \in X \mid \rho_3(x, y, t) < \epsilon\}$ and $O(x, \epsilon, t) = \{y \in X \mid \rho_1(x, y, t) > 1 - \epsilon, \rho_2(x, y, \epsilon) < \epsilon, \rho_3(x, y, t) < \epsilon\}$ as a ball with center x, radius ϵ, and at the time t. Clearly $O_{\rho_1}(x, \epsilon, 0) = O_{\rho_2}(x, \epsilon, 0) = O_{\rho_3}(x, \epsilon, 0) = O(x, \epsilon, 0) = \emptyset$.

Theorem 4.1. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space, $t, t_1, t_2 \in \mathbb{R}^{\geq 0}, x, y \in X$ and $0 < \epsilon, \epsilon_1, \epsilon_2 < 1$. Then*

(i) if $t_1 < t_2$, then $O_{\rho_1}(x, \epsilon, t_1) \subseteq O_{\rho_1}(x, \epsilon, t_2)$;

(ii) if $\epsilon_1 < \epsilon_2$, then $O_{\rho_1}(x, \epsilon_1, t) \subseteq O_{\rho_1}(x, \epsilon_2, t)$;

(iii) if $\epsilon_1 < \epsilon_2$ and $t_1 < t_2$, then $O_{\rho_1}(x, \epsilon_1, t_1) \subseteq O_{\rho_1}(x, \epsilon_2, t_2)$;

(iv) for all $t \in \mathbb{R}^+$, $\{x\} \subseteq O_{\rho_1}(x, \epsilon, t) \neq \emptyset$;

(v) if X is finite, $t \in \mathbb{R}^+$ and $\epsilon_t = (1 - \delta) - \bigvee_{x,y \in X} \rho_1(x, y, t)$, then $O_{\rho_1}(x, \epsilon_t, t) = \{x\}$, where $0 < \delta < 1$;

(vi) if X is finite and $t \in \mathbb{R}^+$, there exists ϵ_{min} such that $O_{\rho_1}(x, \epsilon_{min}, t) = X$.

Proof. (i) Let $y \in O_{\rho_1}(x, \epsilon, t_1)$. Then $\rho_1(x, y, t_1) > 1 - \epsilon$, so by Theorem 3.4, $t_1 < t_2$ implies that $\rho_1(x, y, t_2) > \rho_1(x, y, t_1) > 1 - \epsilon$. Thus $O_{\rho_1}(x, \epsilon, t_1) \subseteq O_{\rho_1}(x, \epsilon, t_2)$.

(ii) Let $y \in O_{\rho_1}(x, \epsilon_1, t)$. Then $\rho_1(x, y, t) > 1 - \epsilon_1$, so $\epsilon_1 < \epsilon_2$ implies that $\rho_1(x, y, t) > 1 - \epsilon_1 > 1 - \epsilon_2$. Thus $O_{\rho_1}(x, \epsilon, t_1) \subseteq O_{\rho_1}(x, \epsilon, t_2)$.

$(iii), (iv)$ It is clear.

(v) Let $y \in O_{\rho_1}(x, \epsilon_t, t)$. Since $\rho_1(x, y, t) \leq \bigvee_{x,y \in X} \rho_1(x, y, t)$, if $\rho_1(x, y, t) > 1 - \epsilon_t$, we get that $\rho_1(x, y, t) = 1$ and so $x = y$.

(vi) Consider $\epsilon_{min} = 1 + \delta - \bigwedge_{x,y \in X} \rho_1(x, y, t)$, where $\bigwedge_{x,y \in X} \rho_1(x, y, t) - \delta > 0$. Since for all $y \in X, \rho_1(x, y, t) > \epsilon_{min}$, we get that $X \subseteq O_{\rho_1}(x, \epsilon_{min}, t)$. □

In a similar way to Theorem 4.1, we have the following Theorem.

Theorem 4.2. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space, $t, t_1, t_2 \in \mathbb{R}^{\geq 0}, x \in X$ and $0 < \epsilon, \epsilon_1, \epsilon_2 < 1$. Then for all $i \in \{2, 3\}$*

- (i) *if $t_1 < t_2$, then $O_{\rho_i}(x, \epsilon, t_1) \subseteq O_{\rho_i}(x, \epsilon, t_2)$;*
- (ii) *if $\epsilon_1 < \epsilon_2$, then $O_{\rho_i}(x, \epsilon_1, t) \subseteq O_{\rho_i}(x, \epsilon_2, t)$;*
- (iii) *if $\epsilon_1 < \epsilon_2$ and $t_1 < t_2$, then $O_{\rho_i}(x, \epsilon_1, t_1) \subseteq O_{\rho_i}(x, \epsilon_2, t_2)$;*
- (iv) *for all $t \in \mathbb{R}^+$, $\{x\} \subseteq O_{\rho_i}(x, \epsilon, t) \neq \emptyset$;*
- (v) *if X is finite, $t \in \mathbb{R}^+$ and $\epsilon'_t = -\delta + \bigwedge\limits_{x,y \in X} \rho_2(x, y, t)$, then $O_{\rho_i}(x, \epsilon'_t, t) = \{x\}$, where $0 < \delta < 1$;*
- (vi) *if X is finite, $t \in \mathbb{R}^+$ and $\epsilon'_{max} = \delta + \bigvee\limits_{x \neq y \in X} \rho_2(x, y, t)$, then $O_{\rho_i}(x, \epsilon'_{max}, t) = X$, where $0 < \delta < 1$.*

Corollary 4.3. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space, $t, t_1, t_2 \in \mathbb{R}^{\geq 0}, x \in X$ and $0 < \epsilon, \epsilon_1, \epsilon_2 < 1$. Then*

- (i) *if $t_1 < t_2$, then $O(x, \epsilon, t_1) \subseteq O(x, \epsilon, t_2)$;*
- (ii) *if $\epsilon_1 < \epsilon_2$, then $O(x, \epsilon_1, t) \subseteq O(x, \epsilon_2, t)$;*
- (iii) *if $\epsilon_1 < \epsilon_2$ and $t_1 < t_2$, then $O(x, \epsilon_1, t_1) \subseteq O(x, \epsilon_2, t_2)$;*
- (iv) *for all $t \in \mathbb{R}^+$, $\{x\} \subseteq O(x, \epsilon, t) \neq \emptyset$;*
- (v) *if X is finite and $t \in \mathbb{R}^+$, then $O(x, \epsilon_t, t) = O(x, \epsilon'_t, t) = \{x\}$;*
- (vi) *if X is finite and $t \in \mathbb{R}^+$, then $O(x, \epsilon_{min}, t) = O(x, \epsilon'_{max}, t) = X$.*

Theorem 4.4. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space, $t \in \mathbb{R}^{\geq 0}, x, y \in X$ and $0 < r < 1$. Then*

- (i) *$x \in O(y, \epsilon, t)$ if and only if $y \in O(x, \epsilon, t)$;*
- (ii) *if $O(x, \epsilon, t) \cap O(y, \epsilon, t) \neq \emptyset$ then for all $2 \leq k \in \mathbb{N}$ we have $O(x, \epsilon, kt) \cap O(y, \epsilon, kt) \neq \emptyset$;*

Proof. It is obvious, by definition. □

In [3], George and Veeramani proved that in any metric space (X, d) (Remark 2.8) the open ball O_{ρ_1} is an open set. In the following theorem we show that the open ball O is an open set.

Theorem 4.5. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space, $t \in \mathbb{R}^{\geq 0}, x \in X$ and $0 < r < 1$. Then for all $i \in \{2, 3\}$*

(*i*) $O(x,\epsilon,t) = O_{\rho_1}(x,\epsilon,t) \cap O_{\rho_2}(x,\epsilon,t) \cap O_{\rho_3}(x,\epsilon,t)$;

(*ii*) $O\rho_i(x,\epsilon,t)$ *is an open set;*

(*iii*) $O(x,\epsilon,t)$ *is an open set.*

Proof. (*i*) It is obvious.

(*ii*) Let $y \in O\rho_i(x,\epsilon,t)$. Then there exists $t > t_0 \in \mathbb{R}^+$ and ϵ' such that $\rho_i(x,y,t_0) < \epsilon$ and $S(\epsilon',\epsilon) < \epsilon$. Consider $B = O\rho_i(y,\epsilon',t-t_0)$. For all $z \in O\rho_i(y,\epsilon',t-t_0)$, we have $\epsilon > S(\epsilon,\epsilon') \geq S(\rho_i(x,y,t_0),\rho_i(y,z,t-t_0)) \geq \rho_i(x,z,t)$. It follows that $y \in B \subseteq O\rho_i(x,\epsilon,t)$ and so $O\rho_i(x,\epsilon,t)$ is an open set.

(*iii*) It is immediate by (*i*), (*ii*) □

Theorem 4.6. *Let* $(X,\rho_1,\rho_2,\rho_3,T,S)$ *be a* KM*-single valued neutrosophic metric space,* $t \in \mathbb{R}^{\geq 0}, x \in X$ *and* $0 < r < 1$. *Then for all* $i \in \{1,2,3\}$

(*i*) *if* X *is a finite set and* $\tau = \{O(x,\epsilon,0), O(x,\epsilon_{max},t) \mid x \in X, 0 < \epsilon < 1, t \in \mathbb{R}^+\}$, *then* (X,τ) *is a topological space;*

(*ii*) $\mathcal{O} = \{O_{\rho_2}(x,\epsilon,t) \mid x \in X, 0 < \epsilon < 1, t \in \mathbb{R}^+\}$ *forms a base of a topology* τ_{ρ_2} *in* X*;*

(*iii*) $\mathcal{O} = \{O_{\rho_3}(x,\epsilon,t) \mid x \in X, 0 < \epsilon < 1, t \in \mathbb{R}^+\}$ *forms a base of a topology* τ_{ρ_3} *in* X*;*

(*iv*) $\mathcal{O} = \{O_{\rho_1}(x,\epsilon,t) \mid x \in X, 0 < \epsilon < 1, t \in \mathbb{R}^+\}$ *forms a base of a topology* τ_{ρ_1} *in* X*.*

Proof. (*i*) It is obvious.

(*ii*), (*iii*) Let $x \in X$ and $i \in \{2,3\}$. Then by Theorem 4.2, for all $0 < \epsilon < 1, t \in \mathbb{R}^+, x \in O_{\rho_i}(x,\epsilon,t)$ and so $X \subseteq \bigcup_{\epsilon,t} O_{\rho_i}(x,\epsilon,t)$. Let for $x,y,z \in X, 0 < \epsilon,\epsilon' < 1, t,t' \in \mathbb{R}^+$, we have $z \in O_{\rho_i}(x,\epsilon,t) \cap O_{\rho_i}(y,\epsilon',t')$. Then $\rho_2(x,z,t) < \epsilon, \rho_2(y,z,t') < \epsilon', \rho_3(x,z,t) < \epsilon$ and $\rho_3(y,z,t') < \epsilon'$. Thus there exists $t_0 \in \mathbb{R}^+$ such that $t_0 < t, t_0 < t'$, which $\rho_i(x,z,t_0) < \epsilon$ and $\rho_i(y,z,t_0) < \epsilon'$. Now consider $0 < \epsilon'' < 1$, $t'' \in \mathbb{R}^+$ such that $T_{min}(\epsilon,\epsilon') > \epsilon''$ and $t'' < T_{min}\{t-t_0,t'-t_0\}$. If $m \in O_{\rho_i}(z,\epsilon'',t'')$, then $\rho_i(m,y,t') \leq S(\rho_i(m,z,t''),\rho_i(z,y,t_0)) < S(\epsilon'',\epsilon') < \epsilon'$ and so $m \in O_{\rho_i}(y,\epsilon',t')$. Analogously, one can see that $O_{\rho_i}(z,\epsilon'',t'') \subseteq O_{\rho_i}(x,\epsilon,t)$.

(*iv*) Let $x \in X$. Then by Theorem 4.1, for all $0 < \epsilon < 1, t \in \mathbb{R}^+, x \in O_{\rho_1}(x,\epsilon,t)$ and so $X \subseteq \bigcup_{\epsilon,t} O_{\rho_1}(x,\epsilon,t)$. Let for $x,y,z \in X, 0 < \epsilon,\epsilon' < 1, t,t' \in \mathbb{R}^+$, we have $z \in O_{\rho_1}(x,\epsilon,t) \cap O_{\rho_1}(y,\epsilon',t')$. Then $\rho_1(x,z,t) > 1-\epsilon$ and $\rho_1(y,z,t') > 1-\epsilon'$. Thus there exists $t_0 \in \mathbb{R}^+$ such that $t_0 < t, t_0 < t'$, which $\rho_1(x,z,t_0) > 1-\epsilon$ and $\rho_1(y,z,t_0) > 1-\epsilon'$. Now consider $0 < \epsilon'' < 1$, $t'' \in \mathbb{R}^+$ such that $T_{min}(\epsilon,\epsilon') > \epsilon''$ and $t'' < T_{min}\{t-t_0,t'-t_0\}$. If $m \in O_{\rho_1}(z,\epsilon'',t'')$, then $\rho_1(m,y,t') \geq T(\rho_1(m,z,t''),\rho_1(z,y,t_0)) > T(1-\epsilon'',1-\epsilon') > 1-\epsilon'$ and so $m \in O_{\rho_1}(y,\epsilon',t')$. Analogously, one can see that $O_{\rho_1}(z,\epsilon'',t'') \subseteq O_{\rho_1}(x,\epsilon,t)$. □

Corollary 4.7. *Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space, $t \in \mathbb{R}^{\geq 0}, x \in X$ and $0 < r < 1$. Then $\mathcal{O} = \{O(x, \epsilon, t) \mid x \in X, 0 < \epsilon < 1, t \in \mathbb{R}^+\}$ forms a base of a topology $\tau_{\rho_1,\rho_2,\rho_3}$ in X.*

5. KM-Single Valued Neutrosophic Metric Spaces and Metric Spaces

In this section, we present the connection between KM-single valued neutrosophic metric spaces and metric spaces with respect to induced equivalence relation based on unite interval values.

Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space and $\alpha, \beta, \gamma \in [0, 1]$. For every time t, on X^2, define $\rho_1^{(\alpha,t)} = \{(x, y) \mid \rho_1(x, y, t) \geq \alpha\}, \rho_2^{(\beta,t)} = \{(x, y) \mid \rho_2(x, y, t) \leq \beta\}$ and $\rho_3^{(\gamma,t)} = \{(x, y) \mid \rho_3(x, y, t) \leq \gamma\}$ as α-part, β-part and γ-part.

Theorem 5.1. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space and $\alpha, \beta, \gamma \in [0, 1]$. Then*

(*i*) *if $t = 0$, then $\rho_1^{(\alpha,t)} = \rho_2^{(\beta,t)} = \rho_3^{(\gamma,t)} = X^2$ if and only if $\alpha = 0$ and $\beta = \gamma = 1$;*

(*ii*) *if $\alpha = \beta = \gamma = 0$, then $\rho_1^{(\alpha,t)} = X^2, \rho_2^{(\beta,t)} = \rho_3^{(\gamma,t)} = \triangle$;*

(*iii*) *if $\alpha = \beta = \gamma = 1$, then $\rho_1^{(\alpha,t)} = \triangle, \rho_2^{(\beta,t)} = \rho_3^{(\gamma,t)} = X^2$;*

(*iv*) *$\rho_1^{(\alpha,t)}, \rho_2^{(\beta,t)}$ and $\rho_3^{(\gamma,t)}$ are reflexive and symmetric relations on $X(t > 0)$;*

(*v*) *$\rho^{(\alpha,\beta,\gamma,t)} = \rho_1^{(\alpha,t)} \cap \rho_2^{(\beta,t)} \cap \rho_3^{(\gamma,t)}$ is a reflexive and symmetric relation on $X(t > 0)$.*

Proof. (*i*) Let $t = 0$ and $\alpha, \beta, \gamma \in [0, 1]$. Then $\rho_1^{(\alpha,t)} = \{(x, y) \mid \rho_1(x, y, 0) \geq \alpha\} = \{(x, y) \mid 0 \geq \alpha\}$. Hence $\rho_1^{(\alpha,t)} = X^2$ if and only if $\alpha = 0$. In a similar way, $\rho_2^{(\beta,t)} = \rho_3^{(\gamma,t)} = X^2$ if and only if $\beta = \gamma = 1$.

(*ii*), (*iii*) Obviously are proved.

(*iv*) Let $\alpha, \beta, \gamma \in [0, 1]$, $x, y, z \in X$ and $t, s \in \mathbb{R}^{\geq 0}$. Since $\rho_1(x, x, t) = 1 \geq \alpha, \rho_2(x, x, t) = 0 \leq \beta$ and $\rho_3(x, x, t) = 0 \leq \gamma$, we get that $\rho_1^{(\alpha,t)}, \rho_2^{(\beta,t)}, \rho_3^{(\gamma,t)}$ are reflexive relations. Clearly $\rho_1^{(\alpha,t)}, \rho_2^{(\beta,t)}, \rho_3^{(\gamma,t)}$ are symmetric relations.

(*v*) It is similar to item (*iv*). □

In the following Example, we describe some applications of KM-single valued neutrosophic metric space. We discuss applications of KM-single valued neutrosophic metric space for studying the competition along with algorithms. The KM-single valued neutrosophic metric space has many utilizations in different areas, where we connect KM-single valued neutrosophic metric space to other sciences such as economics, computer sciences and other engineering sciences. We present an example of application of KM-single valued neutrosophic metric space in optimization in economic.

Example 5.2. (Decision in Economic) Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of all factories of a town and by Table 1 be metrics. We want to combine the performances of these factories in a one year course ($t = 1$) to decide on how they work together. In Table 2, we is extracted a list of energy consumption of factories by ρ_1, a list of the profits of production of factories by ρ_2 and a list of losses of producing of factories by ρ_3. For Example, if energy consumption of factories x_1, x_2 is equal to 60/100, it is denoted by $\rho_1(x_1, x_2, 1) = 0.6$, if losses of producing are equal to 30/100, it is denoted by $\rho_1(x_1, x_2, 1) = 0.3$ and if profits of producing are equal to 20/100, it is denoted by $\rho_1(x_1, x_2, 1) = 0.2$. Clearly $(X, \rho_1, \rho_2, \rho_3, T_{pr}, S_{max})$ is a KM-single valued neutrosophic metric space. So consider a control for energy consumption, profits of producing and losses of producing with factory cooperation by α-part, β-part and γ-part. For $\alpha = 0.3, \beta = 0.2, \gamma = 0.12$, $t = 1$, we obtain $\rho_1^{(\alpha,t)} = X^2 \setminus \{(x_1, x_4), (x_4, x_1)\}, \rho_2^{(\beta,t)} = \triangle \cup \{(x_1, x_2), (x_2, x_1), (x_2, x_3), (x_3, x_2), (x_3, x_4), (x_4, x_3)\} = \rho_3^{(\gamma,t)}$.

Table 1: Metric space (X, d)

d	x_1	x_2	x_3	x_4
x_1	0	1	2	3
x_2	1	0	1	2
x_3	2	1	0	1
x_4	3	2	1	0

Table 2: Fuzzy metric subsets ρ_1, ρ_2, ρ_3 on $X^2 \times \mathbb{R}^{\geq 0}$.

ρ_1	x_1	x_2	x_3	x_4
x_1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
x_2	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{3}$
x_3	$\frac{1}{3}$	$\frac{1}{2}$	1	$\frac{1}{2}$
x_4	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1

,

ρ_2	x_1	x_2	x_3	x_4
x_1	0	$\frac{1}{6}$	$\frac{2}{9}$	$\frac{1}{4}$
x_2	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{2}{9}$
x_3	$\frac{1}{9}$	$\frac{1}{6}$	0	$\frac{1}{6}$
x_4	$\frac{1}{4}$	$\frac{2}{9}$	$\frac{1}{6}$	0

and

ρ_3	x_1	x_2	x_3	x_4
x_1	0	$\frac{1}{10}$	$\frac{2}{15}$	$\frac{3}{20}$
x_2	$\frac{1}{10}$	0	$\frac{1}{10}$	$\frac{2}{15}$
x_3	$\frac{2}{15}$	$\frac{1}{10}$	0	$\frac{1}{10}$
x_4	$\frac{3}{20}$	$\frac{2}{15}$	$\frac{1}{10}$	0

Clearly $\rho_1^{(\alpha,t)}, \rho_2^{(\beta,t)}$ and $\rho_3^{(\gamma,t)}$ are not transitive relations and so $\rho^{(\alpha,\beta,\gamma,t)}$ is not a transitive relation.

Let $\rho_1^{(\alpha,t,*)}, \rho_2^{(\beta,t,*))}, \rho_3^{(\gamma,t,*))}$ and $\rho^{(\alpha,\beta,\gamma,t,*)}$ be the *transitive closure* of $\rho_1^{(\alpha,t)}$, $\rho_2^{(\beta,t))}$, $\rho_3^{(\gamma,t))}$ and $\rho^{(\alpha,\beta,\gamma,t)}$, respectively (the smallest transitive relation in such a way that contains $\rho_1^{(\alpha,t)}, \rho_2^{(\beta,t))}$, $\rho_3^{(\gamma,t))}$ and $\rho^{(\alpha,\beta,\gamma,t)}$, respectively). Then in the following theorem we show that $\rho_1^{(\alpha,t,*)}, \rho_2^{(\beta,t,*))}$, $\rho_3^{(\gamma,t,*))}$ and $\rho^{(\alpha,\beta,\gamma,t,*)}$ are regular relations. Define $X/\rho^{(\alpha,\beta,\gamma,t,*)} = \{\rho^{(\alpha,\beta,\gamma,t,*)}(x,y,t) \mid x,y \in X)\}$ as set of all equivalence class of X on $\rho^{(\alpha,\beta,\gamma,t,*)}$.

Theorem 5.3. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space and $\alpha, \beta, \gamma \in [0,1]$. Then*

(*i*) *if $t = 0$, then $\rho_1^{(\alpha,t,*)} = \rho_1^{(\alpha,t)}, \rho_2^{(\beta,t,*)} = \rho_2^{(\beta,t)}$ and $\rho_3^{(\gamma,t,*)} = \rho_3^{(\gamma,t)}$;*

(*ii*) *if $\alpha = \beta = \gamma = 0$, then $\rho_1^{(\alpha,t,*)} = \rho_1^{(\alpha,t)}, \rho_2^{(\beta,t,*)} = \rho_2^{(\beta,t)}$ and $\rho_3^{(\gamma,t,*)} = \rho_3^{(\gamma,t)}$;*

(*iii*) *if $\alpha = \beta = \gamma = 1$, then $\rho_1^{(\alpha,t,*)} = \rho_1^{(\alpha,t)}, \rho_2^{(\beta,t,*)} = \rho_2^{(\beta,t)}$ and $\rho_3^{(\gamma,t,*)} = \rho_3^{(\gamma,t)}$.*

Proof. (*i*) Let $\alpha, \beta, \gamma \in [0,1]$ and $x, y \in X$. If $t = 0$, then $\rho_1^{(\alpha,t)} \in \{\emptyset, X^2\}, \rho_2^{(\beta,t)} \in \{\emptyset, X^2\}$ and $\rho_3^{(\gamma,t)} \in \{\emptyset, X^2\}$. Thus in any case, $\rho_1^{(\alpha,t,*)} = \rho_1^{(\alpha,t)}, \rho_2^{(\beta,t,*)} = \rho_2^{(\beta,t)}$ and $\rho_3^{(\gamma,t,*)} = \rho_3^{(\gamma,t)}$.

(*ii*) Let $\alpha = \beta = \gamma = 0$. Then by Theorem 5.1, $\rho_1^{(\alpha,t)} = X^2, \rho_2^{(\beta,t)} = \rho_3^{(\gamma,t)} = \triangle$ and so $\rho_1^{(\alpha,t,*)} = \rho_1^{(\alpha,t)}, \rho_2^{(\beta,t,*)} = \rho_2^{(\beta,t)}$ and $\rho_3^{(\gamma,t,*)} = \rho_3^{(\gamma,t)}$.

(*iii*) It is similar to item (*ii*). □

Theorem 5.4. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space and $\alpha, \beta, \gamma \in [0,1]$. Then for all $t \in \mathbb{R}^{\geq 0}$ and $n \in \mathbb{N}$,*

(*i*) *$\rho_1^{(\alpha,t)} \subseteq \rho_1^{(\alpha,nt)}$ and for all $k \geq n, \rho_1^{(\alpha,nt)} \subseteq \rho_1^{(\alpha,kt)}$;*

(*ii*) *$\rho_2^{(\beta,t)} \subseteq \rho_2^{(\beta,nt)}$ and for all $k \geq n, \rho_2^{(\beta,nt)} \subseteq \rho_2^{(\beta,kt)}$;*

(*iii*) *$\rho_3^{(\gamma,t)} \subseteq \rho_3^{(\gamma,nt)}$ and for all $k \geq n, \rho_3^{(\gamma,nt)} \subseteq \rho_3^{(\gamma,kt)}$.*

Proof. (*i*) Let $x, y \in X, t \in \mathbb{R}^{\geq 0}$ and $n \in \mathbb{N}$. If $(x,y) \in \rho_1^{(\alpha,t)}$, then simple induction concludes that $\alpha \leq T_{min}(\rho_1(x,y,t), \underbrace{\rho_1(y,y,t), \ldots, \rho_1(y,y,t)}_{(n-1)\text{-times}}) \leq \rho_1(x,y,nt)$, so $\rho_1^{(\alpha,t)} \subseteq \rho_1^{(\alpha,nt)}$ and for all $k \geq n, \rho_1^{(\alpha,nt)} \subseteq \rho_1^{(\alpha,kt)}$.

(*ii, iii*) Let $x, y \in X, t \in \mathbb{R}^{\geq 0}$ and $n \in \mathbb{N}$. Since $nt > t$, by Theorem 3.4, we get that $\rho_2(x,y,t) \geq \rho_2(x,y,nt), \rho_3(x,y,t) \geq \rho_3(y,y,nt)$ and it concludes that $\rho_2^{(\beta,t)} \subseteq \rho_2^{(\beta,nt)}$, $\rho_3^{(\gamma,t)} \subseteq \rho_3^{(\gamma,nt)}$ and for all $k \geq n, \rho_2^{(\beta,nt)} \subseteq \rho_2^{(\beta,kt)}$ and $\rho_3^{(\gamma,nt)} \subseteq \rho_3^{(\gamma,kt)}$. □

Theorem 5.5. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space and $\alpha, \beta, \gamma \in (0,1)$. Then, for all $t > 0$, there exists the smallest $n \in \mathbb{N}$, such that*

(i) $\rho_1^{(\alpha,t,*)} = \bigcup_{k=1}^{n} \rho_1^{(\alpha,kt)}$, *where for all* $k \geq n$, *we have* $\rho_1^{(\alpha,kt)} = \rho_1^{(\alpha,nt)}$;

(ii) $\rho_2^{(\beta,t,*)} = \bigcup_{k=1}^{n} \rho_2^{(\beta,kt)}$, *where for all* $k \geq n$, *we have* $\rho_2^{(\beta,kt)} = \rho_1^{(\beta,nt)}$;

(iii) $\rho_3^{(\gamma,t,*)} = \bigcup_{k=1}^{n} \rho_3^{(\gamma,kt)}$, *where for all* $k \geq n$, *we have* $\rho_3^{(\gamma,kt)} = \rho_3^{(\gamma,nt)}$.

Proof. Let $n \in \mathbb{N}, \alpha, \beta, \gamma \in (0,1)$ and $x, y \in X$.

(i) By Theorem 5.4, $\rho_1(x,y,t) \leq \rho_1(x,y,nt)$. Since $\lim_{t\to\infty} \rho_1(x,y,t) = 1$, we get that $\lim_{t\to\infty} \rho_1(x,y,nt) = 1$. Thus there exists $N \in \mathbb{N}$ such that for all $t > \lfloor N/n \rfloor$, we have $1-\alpha < \rho_1(x,y,t)$. Hence $\mathcal{A}_\alpha = \{m \in \mathbb{N} \mid \rho_1(x,y,m) \geq \alpha\} \neq \emptyset$ and using well-ordering principle, there exists the smallest $n \in \mathbb{N}$ such that $\rho_1(x,y,nt) \geq \alpha$. By Theorem 5.4, for all $k \geq n, \rho_1^{(\alpha,nt)} \subseteq \rho_1^{(\alpha,kt)}$. In addition, since n is the smallest which $\rho_1(x,y,m) \geq \alpha$, we get that $\rho_1^{(\alpha,kt)} \subseteq \rho_1^{(\alpha,nt)}$ and so for all $k \geq n, \rho_1^{(\alpha,nt)} = \rho_1^{(\alpha,kt)}$. Now, if $(x,y),(y,z) \in \rho_1^{(\alpha,t,*)}$, then, there exists $k,k' \in \mathbb{N}$ in such a way that $(x,y) \in \rho_1^{(\alpha,kt)}$ and $(x,y) \in \rho_1^{(\alpha,k't)}$. Theorem 5.4, implies that $\alpha \leq T_{min}(\rho_1(x,y,t),\rho_1(y,z,t)) \leq T_{min}(\rho_1(x,y,kt),\rho_1(y,z,kt)) \leq \rho_1(x,y,(k+k')t)$. Thus $(x,z) \in \rho_1^{(\alpha,(k+k')t)} \subseteq \bigcup_{k=1}^{n} \rho_1^{(\alpha,kt)} = \rho_1^{(\alpha,t,*)}$ and so $\rho_1^{(\alpha,t,*)}$ is a transitive relation. Suppose that R be a transitive relation such that $\rho_1^{(\alpha,t)} \subseteq R$, by induction we show that $\rho_1^{(\alpha,t,*)} \subseteq R$. It is clear for $n=1$. Assume that it is satisfied for n,if means that $\bigcup_{k=1}^{n} \rho_1^{(\alpha,kt)} \subseteq R$. If $(x,y) \in \bigcup_{k=1}^{n+1} \rho_1^{(\alpha,kt)}$, since $\bigcup_{k=1}^{n+1} \rho_1^{(\alpha,kt)} = \bigcup_{k=1}^{n} \rho_1^{(\alpha,kt)} \cup \rho_1^{(\alpha,t)}$, by assumption of induction we get that $(x,y) \in \bigcup_{k=1}^{n} \rho_1^{(\alpha,kt)} \subseteq R$ or $(x,y) \in \rho_1^{(\alpha,t)} \subseteq R$. It follows that $\rho_1^{(\alpha,t,*)} \subseteq R$ and so it is *transitive closure* of $\rho_1^{(\alpha,t)}$.

In a similar way, one can see that (ii), (iii) are proved. □

Corollary 5.6. *Let* $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ *be a KM-single valued neutrosophic metric space and* $\alpha, \beta, \gamma \in (0,1)$. *Then, for all* $t > 0$, *there exists the smallest* $n \in \mathbb{N}$, *such that* $\rho^{(\alpha,\beta,\gamma,t,*)} = \bigcup_{k=1}^{n} \rho^{(\alpha,\beta,\gamma,nt)}$, *where for all* $k \geq n$, *we have* $\rho^{(\alpha,\beta,\gamma,kt)} = \rho^{(\alpha,\beta,\gamma,nt)}$.

Theorem 5.7. *Let* $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ *be a KM-single valued neutrosophic metric space. Then there exists* $\alpha, \beta, \gamma \in [0,1]$ *such that for all* $t \geq 0$, $\rho^{(\alpha,\beta,\gamma,t,*)} = \rho^{(\alpha,\beta,\gamma,t)}$.

Proof. Let $x, y \in X$. Then consider $\alpha = \bigwedge_{x,y\in X} \rho_1(x,y,t), \beta = \bigvee_{x,y\in X} \rho_1(x,y,t)$ and $\gamma = \bigvee_{x,y\in X} \rho_1(x,y,t)$. Since for all $x, y \in X, \rho_1(x,y,t) \geq \alpha, \rho_2(x,y,t) \leq \beta$ and $\rho_2(x,y,t) \leq \gamma$, for all $t \geq 0$ we ge that $\rho_1^{(\alpha,t,*)} = \rho_1^{(\alpha,t)}, \rho_2^{(\beta,t,*)} = \rho_2^{(\beta,t)}$ and $\rho_3^{(\gamma,t,*)} = \rho_3^{(\gamma,t)}$. It concludes that there exists $\alpha, \beta, \gamma \in [0,1]$ such that for all $t \geq 0$, $\rho^{(\alpha,\beta,\gamma,t,*)} = \rho^{(\alpha,\beta,\gamma,t)}$. □

Example 5.8. Consider the KM-single valued neutrosophic metric space $(X, \rho_1, \rho_2, \rho_3, T, S)$ in Example 5.2. For $\alpha = 0.3, \beta = 0.2, \gamma = 0.12$, by Corollary 5.6, we obtain $n = 2$(we must consider the performances of these factories in two years course $(t = 2)$) and get fuzzy subsets ρ_1, ρ_2, ρ_3 in Table 3. Thus $\rho_1^{(\alpha,2t)} =$

Table 3: Fuzzy metric subsets ρ_1, ρ_2, ρ_3 on $X^2 \times \mathbb{R}^{\geq 0}$.

ρ_1	x_1	x_2	x_3	x_4
x_1	1	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
x_2	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{4}$
x_3	$\frac{1}{4}$	$\frac{1}{3}$	1	$\frac{1}{3}$
x_4	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	1

,

ρ_2	x_1	x_2	x_3	x_4
x_1	0	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{5}$
x_2	$\frac{1}{9}$	0	$\frac{1}{9}$	$\frac{1}{6}$
x_3	$\frac{1}{6}$	$\frac{1}{9}$	0	$\frac{1}{9}$
x_4	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{9}$	0

and

ρ_3	x_1	x_2	x_3	x_4
x_1	0	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{3}{25}$
x_2	$\frac{1}{15}$	0	$\frac{1}{15}$	$\frac{1}{10}$
x_3	$\frac{1}{10}$	$\frac{1}{15}$	0	$\frac{1}{15}$
x_4	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{15}$	0

$\triangle \cup \{(x_1, x_2)\}, \rho_2^{(\beta,2t)} = \rho_3^{(\gamma,2t)} = X^2$ and so $\rho^{(\alpha,\beta,\gamma,2t)} = \triangle \cup \{(x_1, x_2)\}$ is a transitive relation.

From now on, for all KM-single valued neutrosophic metric space $(X, \rho_1, \rho_2, \rho_3, T, S)$, we consider $A, B \in P^*(X)$ are finite subsets.

Definition 5.9. Let $(X, \rho_1, \rho_2, \rho_3, T, S)$ be a KM-single valued neutrosophic metric space, $A, B \in P^*(X), t \in \mathbb{R}^+, x, y \in X$. Define

(*i*) $\overline{\rho_1}(x,B,t) = \bigvee_{y\in B} \rho_1(x,y,t), \overline{\rho_2}(x,B,t) = \bigwedge_{y\in B} \rho_2(x,y,t)$ and $\overline{\rho_3}(x,B,t) = \bigwedge_{y\in B} \rho_3(x,y,t)$.

(*ii*) $\overline{\rho_1}(A,y,t) = \bigvee_{x\in A} \rho_1(x,y,t), \overline{\rho_2}(A,y,t) = \bigwedge_{x\in A} \rho_2(x,y,t)$ and $\overline{\rho_3}(A,y,t) = \bigwedge_{x\in A} \rho_3(x,y,t)$.

(iii) $\overline{\rho_1}(A,B,t) = (\bigwedge_{x\in A}\bigvee_{y\in B}\rho_1(x,y,t)) \wedge (\bigwedge_{y\in B}\bigvee_{x\in A}\rho_1(x,y,t))$,

(iv) $\overline{\rho_2}(A,B,t) = (\bigvee_{x\in A}\bigwedge_{y\in B}\rho_2(x,y,t)) \vee (\bigvee_{y\in B}\bigwedge_{x\in A}\rho_2(x,y,t))$,

(v) $\overline{\rho_3}(A,B,t) = (\bigvee_{x\in A}\bigwedge_{y\in B}\rho_3(x,y,t)) \vee (\bigvee_{y\in B}\bigwedge_{x\in A}\rho_3(x,y,t))$.

In [12], Rodrguez-Lopez and Romaguera, proved that in every GV-fuzzy metric space (X,ρ_1,T), the set of nonempty compact subsets $(\mathcal{K}^*(X))$ of $(X;\tau_X)$ construct a fuzzy metric space as Hausdorf fuzzy metric. In a similar way we have the following Theorem.

Corollary 5.10. *Let $(X,\rho_1,\rho_2,\rho_3,T,S)$ be a KM-single valued neutrosophic metric space, $A,B,C \in \mathcal{K}^*(X), t \in \mathbb{R}^+$. Then $(\mathcal{K}^*(X),\overline{\rho_1},\overline{\rho_2},\overline{\rho_3},T,S)$ is a KM-single valued neutrosophic metric space.*

Theorem 5.11. *Let $(X,\rho_1,\rho_2,\rho_3,T_{min},S_{max})$ be a KM-single valued neutrosophic metric space and $\alpha,\beta,\gamma \in [0,1]$. Then, $(X/\rho^{(\alpha,\beta,\gamma,t,*)},\overline{\rho_1},\overline{\rho_2},\overline{\rho_3},T,S)$ is a KM-single valued neutrosophic metric space.*

Proof. The proof is obtained by Corollary 5.10. □

Definition 5.12. Let $(X,\rho_1,\rho_2,\rho_3,T,S)$ and $(X',\rho'_1,\rho'_2,\rho'_3,T',S')$ be KM-single valued neutrosophic metric spaces. A bijection $\varphi : X \to X'$ is called an isomorphism if for all $x,y \in X$ and for all $i \in \{1,2,3\}$, $\rho_i(x,y,t) = \rho'_i(\varphi(x),\varphi(y),t)$ and denoted it by $(X,\rho_1,\rho_2,\rho_3,T,S) \cong (X',\rho'_1,\rho'_2,\rho'_3,T',S')$.

Corollary 5.13. *Let $(X,\rho_1,\rho_2,\rho_3,T_{min},S_{max})$ be a KM-single valued neutrosophic metric space and $\alpha,\beta,\gamma \in [0,1]$. Then*

(i) *if $t = 0$, then $(X/\rho^{(\alpha,\beta,\gamma,t,*)},\overline{\rho_1},\overline{\rho_2},\overline{\rho_3},T,S) \cong (X,\rho_1,\rho_2,\rho_3,T,S)$;*

(ii) *if $\alpha = \beta = \gamma = 0$, then $(X/\rho^{(\alpha,\beta,\gamma,t,*)},\overline{\rho_1},\overline{\rho_2},\overline{\rho_3},T,S) \cong (X,\rho_1,\rho_2,\rho_3,T,S)$;*

(iii) *if $\alpha = \beta = \gamma = 1$, then $(X/\rho^{(\alpha,\beta,\gamma,t,*)},\overline{\rho_1},\overline{\rho_2},\overline{\rho_3},T,S) \cong (X,\rho_1,\rho_2,\rho_3,T,S)$.*

Definition 5.14. Let $(X,\rho_1,\rho_2,\rho_3,T_{min},S_{max})$ be a KM-single valued neutrosophic metric space and $\alpha,\beta,\gamma \in [0,1]$. For all $x,y \in X$, define

$$d^{(\alpha,t,*)}(x,y) = \begin{cases} 0 & \text{if } x = y \\ |x/\rho_1^{(\alpha,t,*)}| + |y/\rho_1^{(\alpha,t,*)}| & \text{if } x \neq y \end{cases},$$

$$d^{(\beta,t,*)}(x,y) = \begin{cases} 0 & \text{if } x = y \\ |x/\rho_2^{(\beta,t,*)}| + |y/\rho_2^{(\beta,t,*)}| & \text{if } x \neq y \end{cases},$$

$$d^{(\gamma,t,*)}(x,y) = \begin{cases} 0 & \text{if } x = y \\ |x/\rho_3^{(\gamma,t,*)}| + |y/\rho_3^{(\gamma,t,*)}| & \text{if } x \neq y \end{cases},$$

and $d^{(\alpha,\beta,\gamma,t,*)}(x,y) = \begin{cases} 0 & \text{if } x = y \\ |x/\rho^{(\alpha,\beta,\gamma,t,*)}| + |y/\rho^{(\alpha,\beta,\gamma,t,*)}| & \text{if } x \neq y \end{cases}$.

Theorem 5.15. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space and $0 \leq \alpha, \beta, \gamma \leq 1$. Then*

(i) $(X, d^{(\alpha,t,*)})$, $(X, d^{(\beta,t,*)})$, $(X, d^{(\gamma,t,*)})$ *and* $(X, d^{(\alpha,\beta,\gamma,t,*)})$ *are metric spaces.*

(ii) $d^{(\alpha,\beta,\gamma,t,*)} = (d^{(\alpha,t,*)}) \wedge (d^{(\beta,t,*)}) \wedge (d^{(\gamma,t,*)})$.

Proof. It is clear that. □

Let $\mathcal{A} = \{(X, \rho_1, \rho_2, \rho_3, T, S) \mid \rho_1, \rho_2, \rho_3$ are fuzzy subsets on $X^2 \times \mathbb{R}^{\geq 0}\}$ and $\mathcal{B} = \{(X, d) \mid d$ is a metric $\}$. Define $\varphi : \mathcal{A} \to \mathcal{B}$, by $\varphi((X, \rho_1, \rho_2, \rho_3, T, S)) = (X, d^{(\alpha,\beta,\gamma,t,*)})$ based Theorem 5.15 and $\psi : \mathcal{B} \to \mathcal{A}$ by $\varphi((X, d)) = (X, \rho_1, \rho_2, \rho_3, T, S)$ based Corollary 3.9.

Corollary 5.16. *Let $\alpha, \beta, \gamma \in [0, 1]$. Then For sets $\mathcal{A}$ and $\mathcal{B}$, we have a diagram in Figure 1.*

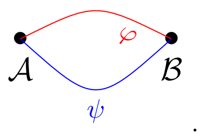

Figure 1: Diagram between KM-single valued neutrosophic metric space and metric space.

5.1. Extended (KM-Single Valued Neutrosophic) Metric

In this subsection, we obtain continuous metrics from KM-single valued neutrosophic metric spaces and continuous KM-single valued neutrosophic metric from metric spaces.

Definition 5.1. Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0, 1)$. For all $x, y \in X$, define $d_\alpha(x, y) = \bigwedge\{t \in \mathbb{R}^+ \mid \rho_1(x, y, t) \geq \alpha\ \}$, $d_\beta(x, y) = \bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x, y, t) \leq \beta\ \}$ and $d_\gamma(x, y) = \bigwedge\{t \in \mathbb{R}^+ \mid \rho_3(x, y, t) \leq \gamma\ \}$.

In what follows, we generate a family of metric spaces from a KM-single valued neutrosophic metric space.

Theorem 5.2. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0, 1)$. Then $(X, d_\alpha), (X, d_\beta)$ and (X, d_γ) are metric spaces.*

Proof. Let $x, y, z \in X$ and $\beta \in (0,1)$. Then $d_\beta(x,x) = \bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x,x,t) \leq \beta\} = \bigwedge\{t \in \mathbb{R}^+ \mid 0 \leq \beta \} = 0$. It is clear that $d_\beta(x,y) = d_\beta(y,x)$. Let $\bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x,y,t) \leq \beta\} = t_0$ and $\bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(y,z,t) \leq \beta\} = s_0$. Thus $\beta = S_{max}(\rho_2(x,y,t_0), \rho_2(y,z,s_0)) \geq \rho_2(x,z,t_0+s_0)$ implies that $\rho_2(x,z,t_0+s_0) \leq \beta$. It follows that $(t_0+s_0) \in \bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x,z,t) \leq \beta\}$ and so $d_\beta(x,y) + d_\beta(y,z) = \bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x,y,t) \leq \beta\} + \bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(y,z,t) \leq \beta\} \geq \bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x,z,t) \leq \beta\} = d_\beta(x,z)$. In a similar way one can see that (X, d_α) and (X, d_γ) are metric spaces. □

Theorem 5.3. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \alpha', \beta, \beta', \gamma, \gamma' \in (0,1)$ and $x, y \in X$. Then*

(i) if $\alpha \leq \alpha'$, then $d_\alpha(x,y) \leq d_{\alpha'}(x,y)$;

(ii) if $\beta \leq \beta'$, then $d_{\beta'}(x,y) \leq d_\beta(x,y)$;

(iii) if $\gamma \leq \gamma'$, then $d_{\gamma'}(x,y) \leq d_\gamma(x,y)$.

Proof. Let $x, y, z \in X$ and $\alpha, \alpha', \beta, \beta', \gamma, \gamma' \in (0,1)$. If $d_\beta(x,y) = t_0$, then $\rho_2(x,y,t_0) \leq \beta \leq \beta'$ and so $t_0 \in \bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x,y,t) \leq \beta'\}$. It concludes that $d_{\beta'}(x,y) \leq t_0 = d_\beta(x,y)$. Other items are proved in a similar way. □

Theorem 5.4. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0,1)$. Then*

$$d_\alpha, d_\beta, d_\gamma : (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \times (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \to \mathbb{R}^{\geq 0},$$

are continuous maps.

Proof. Let $(x_n, y_n)_n$ be a sequence in $X \times X$ that converges to (x,y) with respect to the fuzzy metric $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$. Thus for all $\varepsilon > 0$ and $t \in \mathbb{R}^+$, there exists $N \in \mathbb{N}$, such that for all $n \geq N$, $1 - \epsilon < \rho_1((x_n, x, t)), \rho_2((x_n, x, t)) < \epsilon$ and $\rho_3((x_n, x, t)) < \epsilon$. Hence for all $\alpha, \beta, \gamma, \in (0,1)$, $\{t \in \mathbb{R}^+ \mid \rho_1(x_n, x, t) \geq \alpha\} \neq \emptyset$, $\{t \in \mathbb{R}^+ \mid \rho_2(x_n, x, t) \leq \beta\} \neq \emptyset$ and $\{t \in \mathbb{R}^+ \mid \rho_2(x_n, x, t) \leq \gamma\} \neq \emptyset$. Hence there exists $N \in \mathbb{N}$, such that for all $n \geq N$, and all $\alpha < 1 - \varepsilon, \beta > \varepsilon$ and $\gamma > \varepsilon$,$d_\alpha(x_n, x) < t, d_\beta(x_n, x) < t$ and $d_\gamma(x_n, x) < t$ and so $\lim\limits_{n \to \infty} d_\alpha(x_n, x) = d_\beta(x_n, x) = d_\gamma(x_n, x) = 0$. In a similar way, $\lim\limits_{n \to \infty} d_\alpha(y_n, y) = d_\beta(y_n, y) = d_\gamma(y_n, y) = 0$. Since for all, $n \in \mathbb{N}, d_\alpha(x_n, x) + d_\alpha(x_n, y_n) + d_\alpha(y_n, y) \geq d_\alpha(x,y)$, we get that $|\lim\limits_{n \to \infty} d_\alpha(x_n, y_n) - d_\alpha(x,y)| \leq (\lim\limits_{n \to \infty} d_\alpha(x_n, x) + \lim\limits_{n \to \infty} d_\alpha(y, y_n)) = 0$. In a similar way, $\lim\limits_{n \to \infty} d_\beta(x_n, y_n) = d_\beta(x,y)$ and $\lim\limits_{n \to \infty} d_\gamma(x_n, y_n) = d_\gamma(x,y)$. □

Theorem 5.5. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \alpha', \beta,$*

*$\beta', \gamma, \gamma' \in (0,1)$ and $x, y \in X$. Then a sequence $(x_n)_n$ is a Cauchy sequence in $(X, \rho_1, \rho_2, \rho_3, T_{min}$
$, S_{max})$ if and only if it is a Cauchy sequence in $(X, d_\alpha), (X, d_\beta)$ and (X, d_γ).*

Proof. Suppose $(x_n)_n$ is a Cauchy sequence in metric spaces $(X, d_\alpha), (X, d_\beta)$ and (X, d_γ). Fixed $\varepsilon > 0$. Since $(x_n)_n$ is a Cauchy sequence in (X, d_β), there exists $N \in \mathbb{N}$ such that for all $n, m \geq N, d_\beta(x_n, x_m) < \varepsilon$. It follows that $t_0 = \bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x_n, x_m, t) \leq \beta\} < \varepsilon$ and so $\rho_2(x_n, x_m, t) \leq \rho_2(x_n, x_m, t_0) < \varepsilon$. In a similar way, one can see that $\rho_3(x_n, x_m, t) < \varepsilon$ and $\rho_1(x_n, x_m, t) > 1 - \varepsilon$ and so sequence $(x_n)_n$ is a Cauchy sequence in KM-single valued neutrosophic metric space $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$.

Conversely, let sequence $(x_n)_n$ is a Cauchy sequence in $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ and Fixed $\varepsilon > 0$. Thus for all $t \in \mathbb{R}^+$, there exists $N \in \mathbb{N}$ such that for any $m, n \geq N$, we have

$$1 - \varepsilon < \rho_1(x_n, x_m, t), \rho_2(x_n, x_m, t) < \varepsilon \text{ and } \rho_3(x_n, x_m, t) < \varepsilon.$$

Hence for all $\alpha, \beta, \gamma, \in (0,1)$, $\{t \in \mathbb{R}^+ \mid \rho_1(x_n, x_m, t) \geq \alpha\} \neq \emptyset, \{t \in \mathbb{R}^+ \mid \rho_2(x_n, x_m, t) \leq \beta\} \neq \emptyset$ and $\{t \in \mathbb{R}^+ \mid \rho_2(x_n, x_m, t) \leq \gamma\} \neq \emptyset$. So there exists $N \in \mathbb{N}$ such that for all $n, m \geq N$ and all $\alpha < 1 - \varepsilon, \beta > \varepsilon$ and $\gamma > \varepsilon$, we have $d_\alpha(x_n, x_m) \leq t < \varepsilon, d_\beta(x_n, x_m) < t < \varepsilon$ and $d_\gamma(x_n, x_m) < t < \varepsilon$. It is concluded that the sequence $(x_n)_n$ is a Cauchy sequence in metric spaces $(X, d_\alpha), (X, d_\beta)$ and (X, d_γ). □

Definition 5.6. Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0,1)$. For all $x \in X$ and $k \in \mathbb{N}$, define $\overline{d}_\alpha(x, y) = \dfrac{kd_\alpha(x, y)}{1 + kd_\alpha(x, y)}, \overline{d}_\beta(x, y) = \dfrac{kd_\beta(x, y)}{1 + kd_\beta(x, y)}$ and $\overline{d}_\gamma(x, y) = \dfrac{kd_\gamma(x, y)}{1 + kd_\gamma(x, y)}$.

Theorem 5.7. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0,1)$. Then $(X, \overline{d}_\alpha), (X, \overline{d}_\beta)$ and $(X, \overline{d}_\gamma)$ are metric spaces.*

Proof. The proof is similar to Theorem 5.2. □

Theorem 5.8. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0,1)$. Then*

$$\overline{d}_\alpha, \overline{d}_\beta, \overline{d}_\gamma : (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \times (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \to \mathbb{R}^{\geq 0},$$

are continuous maps.

Proof. The proof is similar to Theorem 5.4. □

Definition 5.9. Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0,1)$ and $\emptyset \neq A \subseteq X$. For all $x \in X$, define $d_\alpha(x, A) = \bigwedge\{d_\alpha(x, y) \mid y \in A\}, d_\beta(x, A) = \bigwedge\{d_\beta(x, y) \mid y \in A\}, d_\gamma(x, A) = \bigwedge\{d_\gamma(x, y) \mid y \in A\}$. In a similar way $\overline{d}_\alpha(x, A), \overline{d}_\beta(x, A)$ and $\overline{d}_\gamma(x, A)$ are defined.

Theorem 5.10. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0, 1)$ and $\emptyset \neq A \subseteq X$. Then for all $x, y \in X$*

(i) *if $x \in A$, then $d_\alpha(x, A) = d_\beta(x, A) = d_\gamma(x, A) = 0$;*

(ii) *for all $a \in A$, $d_\alpha(x, A) \leq d_\alpha(x, a), d_\beta(x, A) \leq d_\beta(x, a), d_\gamma(x, A) \leq d_\gamma(x, a)$;*

(iii) *$|d_\alpha(x, A) - d_\alpha(y, A)| \leq d_\alpha(x, y)$;*

(iv) *$|d_\beta(x, A) - d_\beta(y, A)| \leq d_\beta(x, y)$;*

(v) *$|d_\gamma(x, A) - d_\gamma(y, A)| \leq d_\gamma(x, y)$.*

Proof. $(i), (ii)$ It is clear.

$(iii), (iv), (v)$ Let $x, y \in X$. If $x = y$ or $x, y \in A$, then by items $(i), (ii)$ it is straightforward. Suppose that $x \neq y$. Without loss of generality $x \in A$ and $y \notin A$, implies that $|d_\alpha(x, A) - d_\alpha(y, A)| = d_\alpha(y, A) \leq d_\alpha(y, x)$. If $x, y \notin A$, then there exists $a, a' \in A$ such that $d_\alpha(x, A) = d_\alpha(x, a)$ and $d_\alpha(y, A) = d_\alpha(y, a')$. Since $d_\alpha(x, a) \leq d_\alpha(x, a')$ and $d_\alpha(y, a') \leq d_\alpha(y, a)$, we get that $|d_\alpha(x, a) - d_\alpha(y, a')| \leq |d_\alpha(x, a') - d_\alpha(y, a')| \leq d_\alpha(x, y)$. It follows that for all $x, y \in X, |d_\alpha(x, A) - d_\alpha(y, A)| \leq d_\alpha(x, y), |d_\beta(x, A) - d_\beta(y, A)| \leq d_\alpha(x, y)$ and $|d_\gamma(x, A) - d_\gamma(y, A)| \leq d_\gamma(x, y)$. □

Corollary 5.11. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\beta, \gamma \in (0, 1)$ and $\emptyset \neq A \subseteq X$. Then for all $x, y \in X$*

(i) *if $x \in A$, then $\overline{d}_\alpha(x, A) = \overline{d}_\beta(x, A) = \overline{d}_\gamma(x, A) = 0$;*

(ii) *for all $a \in A$, $\overline{d}_\alpha(x, A) \vee \overline{d}_\beta(x, A) \vee \overline{d}_\gamma(x, A) \leq \overline{d}_\alpha(x, a) \wedge \overline{d}_\beta(x, a) \wedge \overline{d}_\gamma(x, a)$;*

(iii) *$|\overline{d}_\alpha(x, A) - \overline{d}_\alpha(y, A)| \leq \overline{d}_\alpha(x, y)$.*

(iv) *$|\overline{d}_\beta(x, A) - \overline{d}_\beta(y, A)| \leq \overline{d}_\beta(x, y)$;*

(v) *$|\overline{d}_\gamma(x, A) - \overline{d}_\gamma(y, A)| \leq \overline{d}_\gamma(x, y)$.*

Theorem 5.12. *Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0, 1)$. Then for all $x \in X$ and $\emptyset \neq A \subseteq X$, $d_\alpha(x, A), d_\alpha(x, A), d_\alpha(x, A) : (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \times (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \to \mathbb{R}^{\geq 0}$ are continuous maps.*

Proof. Suppose $(x_n)_n$ is a sequence in X and in such a way that for all $t \in \mathbb{R}^+$, $\lim_{n\to\infty} \rho_2(x_n, x, t) = 1$ and $x \in X$. Fixed $\varepsilon > 0$. Thus there exists $N \in \mathbb{N}$ such that for all $n \geq N, \rho_2(x_n, x, t) < \varepsilon$. It follows that for all $\varepsilon < \beta$, we have $\bigwedge\{t \in \mathbb{R}^+ \mid \rho_2(x_n, x, t) \leq \beta\} \neq \emptyset$ and so $d_\beta(x_n, x) < t$. Hence $\lim_{n\to\infty} d_\beta(x_n, x) = 0$. Using Theorem 5.10, $|d_\beta(x, A) - d_\beta(x_n, A)| \leq d_\beta(x, x_n)$. Consequently, $\lim_{n\to\infty} d_\beta(x_n, A) = d_\beta(x, A)$. In a similar way, one can see that $d_\alpha(x, A), d_\alpha(x, A) : (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \times (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \to \mathbb{R}^{\geq 0}$ are continuous maps. □

Corollary 5.13. *Let* $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ *be a* KM*-single valued neutrosophic metric space,* $\alpha\beta, \gamma \in (0,1)$. *Then for all* $x \in X$ *and* $\emptyset \neq A \subseteq X$, $\overline{d}_\alpha(x,A), \overline{d}_\alpha(x,A), \overline{d}_\alpha(x,A) : (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \times (X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max}) \to \mathbb{R}^{\geq 0}$ *are continuous maps.*

Let $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ be a KM-single valued neutrosophic metric space, $\alpha, \beta, \gamma \in (0,1)$. Then for all $x \in X$ and $\emptyset \neq A \subseteq X$, define $\overline{A} = \{y \mid \overline{\rho_1}(y, A, t) = 1, \overline{\rho_2}(y, A, t) = \overline{\rho_3}(y, A, t) = 0 \text{ for all } t > 0\}$ and $\underline{A} = \{y \mid \overline{d}_\alpha(y, A) = \overline{d}_\beta(y, A) = \overline{d}_\gamma(y, A) = 0\}$.

Corollary 5.14. *Let* $(X, \rho_1, \rho_2, \rho_3, T_{min}, S_{max})$ *be a* KM*-single valued neutrosophic metric space,* $\alpha, \beta, \gamma \in (0,1)$. *Then for all* $x \in X$ *and* $\emptyset \neq A \subseteq X$, $x \in \overline{A}$ *if and only if* $x \in \underline{A}$.

6. Conclusion

The present study has introduced a novel concept fuzzy algebra as KM-single valued neutrosophic metric spaces and has constructed finite or infinite KM-single valued neutrosophic metric spaces based on induced unit interval values. We can make a correspondence between metric spaces and KM-single valued neutrosophic metric spaces, so we show that KM-single valued neutrosophic metric spaces generate some topological spaces and metric spaces.

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