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The Role of Ordinary Bessel and Hankel Functions in Simulation of Plasma Valve Mechanism in a Loss-Free Metallic Cylindrical Waveguide

Asghar Karamian, Bahram Jazi ⋆ *and Samaneh Najari*

Abstract

In this paper, a finite cylindrical plasma waveguide is investigated as a plasma valve in the path of a non-dissipative cylindrical waveguide with metal walls. Theoretical simulation to investigate the effect of the main parameters of this plasma valve on the transmission coefficients and reflection coefficients of the symmetric modes is the main part of this paper. The transmittance coefficients of electromagnetic waves in each symmetric mode are introduced in terms of Henkel and ordinary Bessel functions, and the role of these functions in the purification of some modes is investigated. Taking into account the boundary conditions, the transmission coefficient of the output wave modes from the plasma valve are obtained. The diagrams of the mentioned coefficient versus the incident wave frequency, geometry dimensions and the type of the used plasma in the valve are studied.

Keywords: electromagnetic wave, transmission coefficients, plasma frequency, mode matching.

2020 Mathematics Subject Classification: 78A02, 34A06, 41A05.

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1. Introduction

As is well known, the Fabry-Perot resonators are devices in optical structures for purifying or filtering electromagnetic waves in a specific frequency rang [9, 20, 21, 23, 24]. In general, the equations governing the behavior of electromagnetic waves, in addition to being the answer to Maxwell's basic equations, must also accurately meet the boundary conditions in the configuration. Applying these boundary conditions creates quantities in the wave scattering equations in certain configuration [10, 18]. As we know, depending on how many degrees of freedom exist in the equations of an electromagnetic structure, the number of quanta index in their dispersion equations varies. Fabry-Perot resonators are commonly used in theoretical optics topics as well as in textbooks with a single degree of freedom. This results in only one quantum number in the dispersion equations of those systems [5, 8]. Targeted passage of electromagnetic waves in bounded systems such as waveguides is a familiar subject for which researchers have conducted numerous theoretical studies and practical experiments [19, 22].

As we know, one of the causes of return and reflection waves in waveguide systems is the appearance of mismatch of wave impedance in the waveguide. Analysis of the behavior of electromagnetic waves and their transmission and reflection from one environment to another is strongly related to a parameter change called wave impedance in the waveguide [7, 16]. Recently, this issue has led to the use of impedance matching theory in theoretical studies on the passage and reflection of waves in different waveguides by researchers [6, 11, 12, 13, 14]. In the mentioned studies, each injector and the receiver media of the electromagnetic waves have been considered semi bounded. The presence of a lamb from a special waveguide (cylindrical plasma resonator) at the confluence of two semi-bounded cylindrical waveguides, corrects and even adjusts the behavior of the transmitting and reflecting waves in these waveguides. In other words, in this case, the plasma resonator can play the role of an electromagnetic valve to pass or not pass electromagnetic waves from one waveguide to another waveguide, in certain modes. The influence of the geometric dimensions and properties of the constituents of the plasma valve on the coefficient of transmission and reflection is an issue that we will address in this article.

The present work has been organized in four sections where the introduction was presented as Section 1. In Section 2, the configuration of the plasma valve and the governing equations are introduced. In this section, the equations of the waves dispersion and the reflection and transmission coefficients of the reflected and transmitted waves are presented. In Section 3, the graphs of the transmission coefficients of the output wave from plasma valve versus the geometrical parameters and properties of the used plasma are investigated. Finally, a summary and conclusion are presented in Section 4. In the end of the paper, the governed law on energy conservation is described as the appendix section.

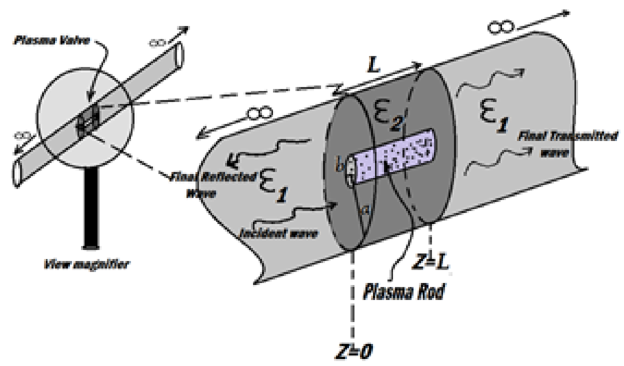

Figure 1: Configuration and geometric dimensions of waveguide and plasma valve.

2. The Considered Configurations and Governed Equations

The structure consists of a plasma resonator between two semi-bounded cylindrical waveguides with a circular cross section with radius a. The two semi-bounded waveguides are simply filled with a non-dissipative dielectric constant material with dielectric permittivity ε_1. The plasma resonator consists of a plasma rod with radius b of length L, which is enclosed in a dielectric layer with dielectric permittivity ε_2. Also, the radius of the plasma rod (the region $0 < z < L$) is considered to be much smaller than the radius of the waveguide ($b << a$) as shown in Figure 1. The landing wave is sent from the left side of the plasma valve. By solving the wave equation, the z component of the potential vector of the incident wave with its quantum number (j) is obtained as follows [15]:

$$A^i_{zj} = J_0(\frac{\gamma_j}{a}\rho)e^{ik^i_{zj}Z}, \tag{1}$$

where J_0 is the Bessel function of the first kind of zero order, γ_j is the j-th zero of the Bessel function of the zero order and k^i_{zj} is propagation constant of the incident wave, in which

$$(k^i_{zj})^2 = (\Gamma_{d1})^2 - (\frac{\gamma_j}{a})^2,$$
$$\Gamma_{d1} = \omega\sqrt{\epsilon_1\mu_0},$$

where ω is the frequency of the incident wave. The incident wave is reflected and transmitted from the first boundary of the plasma valve at $z = 0$. Based on mode matching technique, the reflected and transmitted waves are considered as an infinite series of Eigen modes with quantization m and n, respectively [4, 17].

Be noted that the reflected and transmitted waves are in symmetric mode the same as the incident wave. Therefore, the m and n are modes number of the reflected and transmitted waves (for example TM_{0m} and TM_{0n}). The same as the incident wave, the potential vector Az of the reflected wave from the first boundary ($z = 0$) can be written as [15]:

$$A^r_{zm} = R_m J_0(\frac{\gamma_m}{a}\rho)e^{-ik^r_{zm}Z},$$

where R_m is the reflection coefficient of reflected wave and k^r_{zm} is the propagation constant of the reflected wave in the region $z < 0$. Also, the potential vector A_z of transmitted wave from the first boundary ($z = 0$) are obtained as [15]:

$$A^t_{zn} = \sum_{n=1}^{\infty} T_n \begin{cases} J_0(\frac{\beta_{pn}}{a}\rho)e^{ik^t_{zpn}Z} & 0 < \rho < b, \\ (x_1(n)H_0^{(1)}(\frac{\beta_{dn}}{a}\rho) + x_2(n)J_0(\frac{\beta_{dn}}{a}\rho))e^{ik^t_{zdn}Z} & b < \rho < a, \end{cases}$$

where $H_0^{(1)}$ is the Hankel function of the first kind of zero order, β_{pn} , β_{dn}, $x_1(n)$ and $x_2(n)$ are the constant coefficients of the wave equation, k^t_{zpn} and k^t_{zdn} are the propagation constants of the transmitted wave in the plasma and dielectric in the plasma resonator, respectively. Note that, the propagation constants of the transmitted wave in the plasma and dielectric region are equal. It means that:

$$(k^t_{zdn})^2 = (\Gamma_{d2})^2 - (\frac{\beta_{dn}}{a})^2 = (k^t_{zpn})^2 = (\Gamma_p)^2 - (\frac{\beta_{pn}}{a})^2. \quad (2)$$

The transmitted wave is propagated in the region $0 < Z < L$ and is reflected and transmitted in the second boundary ($z = L$), again. Therefore, the potential vector A_z of the reflected and transmitted wave with r'_m (reflection coefficient) and t'_n (transmission coefficient) can be written respectively, as follows:

$$A^t_{zm'} = \sum_{m'=1}^{\infty} r'_m \begin{cases} J_0(\frac{\beta_{pm'}}{a}\rho)e^{-ik^{rp}_{zpm'}Z} & 0 < \rho < b, \\ (x_1(m')H_0^{(1)}(\frac{\beta_{dm'}}{a}\rho) + x_2(m')J_0(\frac{\beta_{dm'}}{a}\rho))e^{-ik^{rd}_{zdm'}Z} & b < \rho < a, \end{cases}$$

$$A^t_{zn'} = \sum_{n'=1}^{\infty} t'_n J_0(\frac{\gamma'_n}{a}\rho)e^{ik^t_{zn'}Z},$$

where

$$(k^{rp}_{zpm'})^2 = (\Gamma_p)^2 - (\frac{\beta_{pm'}}{a})^2 = (k^{rd}_{zdm'})^2 = (\Gamma_{d2})^2 - (\frac{\beta_{dm'}}{a})^2.$$

In the above equations, n' and m' are the quantization of the reflected and transmission wave from the second boundary $z = L$, respectively. In continue, by using the boundary conditions in $\rho = a$ and $\rho = b$ in the region $0 < Z < L$ the constant coefficients in the mentioned equations will be determined.

By using the continuity of the components of electric and magnetic fields on the boundary surface between the plasma and dielectric $\rho = b$ in the region $0 < Z < L$, the coefficients $x_1(n)$, $x_2(n)$, $x_1(m')$ and $x_2(m')$ in the above equations can be obtained as follows [7, 12]:

$$x_2(n) = -x_1(n)\frac{H_0{}^{(1)}(\beta_{dn})}{J_0(\beta_{dn})}, \tag{3}$$

$$x_2(m') = -x_1(m')\frac{H_0{}^{(1)}(\beta_{dm'})}{J_0(\beta_{dm'})},$$

$$x_1(n) = \frac{\beta_{pn}}{\beta_{dn}}\frac{J_1(\beta_{pn}\frac{b}{a})J_0(\beta_{dn})}{H_1^{(1)}(\beta_{dn}\frac{b}{a})J_0(\beta_{dn}) - H_0^{(1)}(\beta_{dn})J_1(\beta_{dn}\frac{b}{a})},$$

$$x_1(m') = \frac{\beta_{pm'}}{\beta_{dm'}}\frac{J_1(\beta_{pm'}\frac{b}{a})J_0(\beta_{dm'})}{H_1^{(1)}(\beta_{dn}\frac{b}{a})J_0(\beta_{dm'}) - H_0^{(1)}(\beta_{dm'})J_1(\beta_{dm'}\frac{b}{a})},$$

where $H_1^{(1)}$ and J_1 are the Hankel function of the first kind of order one and Bessel function of order one, respectively.

Also, in this region $(0 < Z < L)$, the dispersion equation of the transmitted wave from $Z = 0$ and reflected wave from $Z = L$ are, respectively:

$$\frac{\varepsilon_{d2}}{\varepsilon_p}\frac{J_0(\beta_{pn}\frac{b}{a})}{H_0^{(1)}(\beta_{dn}\frac{b}{a})J_0(\beta_{dn}) - H_0^{(1)}(\beta_{dn})J_0(\beta_{dn}\frac{b}{a})} = \frac{\beta_{dn}}{\beta_{pn}}\frac{J_1(\beta_{pn}\frac{b}{a})}{H_1^{(1)}(\beta_{dn}\frac{b}{a})J_0(\beta_{dn}) - H_0^{(1)}(\beta_{dn})J_1(\beta_{dn}\frac{b}{a})}, \tag{4}$$

$$\frac{\varepsilon_{d2}}{\varepsilon_p}\frac{J_0(\beta_{pm'}\frac{b}{a})}{H_0^{(1)}(\beta_{dm'}\frac{b}{a})J_0(\beta_{dm'}) - H_0^{(1)}(\beta_{dm'})J_0(\beta_{dm'}\frac{b}{a})} = \frac{\beta_{dm'}}{\beta_{pm'}}\frac{J_1(\beta_{pm'}\frac{b}{a})}{H_1^{(1)}(\beta_{dm'}\frac{b}{a})J_0(\beta_{dm'}) - H_0^{(1)}(\beta_{dm'})J_1(\beta_{dm'}\frac{b}{a})}, \tag{5}$$

where $\varepsilon_p = \varepsilon_0(1 - \frac{\omega_p^2}{\omega^2})$ is the permittivity constants of the plasma in the cold and collision less approximation. It must be mentioned that ω is the wave frequency and ω_p is the electron plasma frequency [1].

In continue, by using the Equations (2), (3), (4) and (5), the coefficients β_{pn}, β_{dn}, $\beta_{pm'}$ and $\beta_{dm'}$ can be determined. It must be noted, to obtain these coefficients the condition $b << a$ is used. Therefore, by extending the Equations (2), (3), (4) and (5) in powers of $\frac{b}{a}$ and equate the terms of the same orders, the mentioned coefficients can be obtained [12], as

$$\beta_{d\begin{pmatrix} n \\ m' \end{pmatrix}} = \begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix} + \frac{\pi}{4}\frac{N_0(\begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix})}{J_1(\begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix})}\frac{\varepsilon_{d2} - \varepsilon_p}{\varepsilon_{d2}}(\begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix})^2(\frac{b}{a})^2,$$

$$\beta_{p\begin{pmatrix} n \\ m' \end{pmatrix}} = \sqrt{(a\Gamma_p)^2 - (a\Gamma_{d2})^2 + (\begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix})^2}$$
$$+ \frac{\pi}{4} \frac{N_0(\begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix})}{J_1(\begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix})} \frac{\varepsilon_{d2} - \varepsilon_p}{\varepsilon_{d2}} \frac{(\begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix})^3}{\sqrt{(a\Gamma_p) - (a\Gamma_{d2})^2 + (\begin{pmatrix} \gamma_n \\ \gamma_{m'} \end{pmatrix})^2}} (\frac{b}{a})^2.$$

Finally, by applying the continuity condition of the components of electric and magnetic fields on the boundaries $z = 0$ and $z = L$ and using orthogonality of the Bessel functions, the transmission and reflection coefficients of the reflected and transmitted wave from boundaries $z = 0$ and $z = L$ can be obtained in the following form [2, 3]:

$$R_m = -\frac{\gamma_j}{\gamma_m}\delta_{jm} + \frac{2}{(J_1(\gamma_m))^2(\gamma_m)} \Big[\sum_{n=1}^{\infty} T_n(A_{nm}) + \sum_{m'=1}^{\infty} r_{m'}(A_{mm'})\Big],$$
$$t'_n = \frac{2}{(J_1(\gamma_{n'}))^2(\gamma_{n'})} \Big[\sum_{n=1}^{\infty} T_n(g_{nn'}) + \sum_{m'=1}^{\infty} r_{m'},(H_{m'n'})\Big],$$
$$k^r_{zm}(J_1(\gamma_m))^2\gamma_m\delta_{jm} = \sum_{n=1}^{\infty} T_n(F_{nm}) + \sum_{m'=1}^{\infty} r_{m'}(E_{m'm}),$$
$$\sum_{n=1}^{\infty} T_n(O_{nn'}) = \sum_{m'=1}^{\infty} r_{m'}(P_{m'n'}), \tag{6}$$

where

$$A_{mn} = \frac{\beta_{pn}}{a}\int_0^b J_1(\beta_{pn}\frac{\rho}{a})J_1(\gamma_m\frac{\rho}{a})\rho \mathrm{d}\rho$$
$$+ \int_b^a \Big(x_1(n)H_1^{(1)}(\frac{\beta_{dn}}{a}\rho) + x_2(n)J_1(\frac{\beta_{dn}}{a}\rho)\Big) \times J_1(\gamma_m\frac{\rho}{a})\rho \mathrm{d}\rho,$$
$$F_{nm} = \Big[\frac{\beta_{pn}}{a}\int_0^b J_1(\beta_{pn}\frac{\rho}{a})J_1(\gamma_m\frac{\rho}{a})\rho \mathrm{d}\rho\Big] \Big[k^r_{zm} + k^t_{zn}\frac{\varepsilon_{d1}}{\varepsilon_p}\Big]$$
$$+ \Big[\frac{\beta_{dn}}{a}\int_b^a \Big(x_1(n)H_1^{(1)}(\frac{\beta_{dn}}{a}\rho) + x_2(n)J_1(\frac{\beta_{dn}}{a}\rho)\Big) \times J_1(\gamma_m\frac{\rho}{a})\rho \mathrm{d}\rho\Big]$$
$$\Big[k^r_{zm} + k^t_{zn}\frac{\varepsilon_p}{\varepsilon_{d2}}\Big],$$
$$g_{nn'} = \Big[\frac{\beta_{pn}}{a}\int_0^b J_1(\beta_{pn}\frac{\rho}{a})J_1(\gamma_{n'}\frac{\rho}{a})\rho \mathrm{d}\rho$$
$$+ \frac{\beta_{dn}}{a}\int_b^a \Big(x_1(n)H_1^{(1)}(\frac{\beta_{dn}}{a}\rho) + x_2(n)J_1(\frac{\beta_{dn}}{a}\rho)\Big) \times J_1(\gamma'_n\frac{\rho}{a})\rho \mathrm{d}\rho\Big]$$

$$e^{i(k^t_{zn}-k^t_{zn'})L},$$

$$H_{m'n'} = \Big[\frac{\beta_{pm'}}{a}\int_0^b J_1(\beta_{pm'}\frac{\rho}{a})J_1(\gamma_{n'}\frac{\rho}{a})\rho \mathrm{d}\rho + \frac{\beta_{dm'}}{a}\int_b^a \Big(x_1(m')H_1^{(1)}(\frac{\beta_{dm'}}{a}\rho)+x_2(m')J_1(\frac{\beta_{dm'}}{a}\rho)\Big)\times J_1(\gamma'_n\frac{\rho}{a})\rho \mathrm{d}\rho\Big] e^{-i(k^t_{zm'}-k^t_{zn'})L},$$

$$O_{nn'} = \Big(\Big[\frac{\beta_{pn}}{a}\int_0^b J_1(\beta_{pn}\frac{\rho}{a})J_1(\gamma_{n'}\frac{\rho}{a})\rho \mathrm{d}\rho\Big]\Big[(\frac{\varepsilon_{d1}}{\varepsilon_{d2}})k^r_{zn}+k^t_{zn'})\Big] + \Big[\frac{\beta_{dn}}{a}\int_b^a \Big(x_1(n)H_1^{(1)}(\frac{\beta_{dn}}{a}\rho)+x_2(n)J_1(\frac{\beta_{dn}}{a}\rho)\Big)\times J_1(\gamma'_n\frac{\rho}{a})\rho \mathrm{d}\rho\Big] \Big[(\frac{\varepsilon_{d1}}{\varepsilon_{d2}})k^r_{zn}+k^t_{zn'}\Big]\Big)\times e^{i(k^t_{zn}-k^t_{zn'})L},$$

$$P_{m'n'} = \Big(\Big[\frac{\beta_{pm'}}{a}\int_0^b J_1(\beta_{pm'}\frac{\rho}{a})J_1(\gamma_{n'}\frac{\rho}{a})\rho \mathrm{d}\rho\Big]\Big[(\frac{\varepsilon_{d1}}{\varepsilon_p})k^r_{zm'}+k^t_{zn'})\Big] + \Big[\frac{\beta_{dm'}}{a}\int_b^a \Big(x_1(m')H_1^{(1)}(\frac{\beta_{dm'}}{a}\rho)+x_2(m')J_1(\frac{\beta_{dm'}}{a}\rho)\Big)\times J_1(\gamma'_n\frac{\rho}{a})\rho \mathrm{d}\rho\Big] \Big[(\frac{\varepsilon_{d1}}{\varepsilon_p})k^r_{zm'}+k^t_{zn'}\Big]\Big)\times e^{i(k^t_{zm'}-k^t_{zn'})L}.$$

By solving the set of the coupled Equation (6), the reflection and transmission coefficients of the reflected and transmitted waves from $z = 0$ and $z = L$ are obtained. The results of solving the above equations report that the mentioned coefficients are the function of geometry dimensions, incident wave frequency and type of the plasma in the plasma valve. Because of the transverse anisotropy in this problem to apply the boundary conditions, the induced dipoles on the dielectric-plasma interface cause radiation to propagate in either direction. Therefore, the mathematical relation of a field in transient and reflective waves cannot contain only one function. As mentioned in the manuscript, the reflected and transmitted waves are considered as an infinite series of eigen modes based on mode matching technique. Since, the solutions of the wave equations for the reflected and transmitted waves are obtained as specific functions with different arguments, therefore for one incident mode in quantum j, the existence of only one reflected mode and one transmission mode cannot be satisfied the boundary conditions on $z = 0$ and $z = d$. Therefore, the presence of other mode must be necessary to satisfy the boundary conditions.

The transmission coefficients of the final transmitted wave from the plasma valve (transmitted waves from the boundary $z = L$) have an important role in control and filtering of the waves in this instrument. In continuum the dependence of this coefficient to the properties of the structure, will be investigated in the next section.

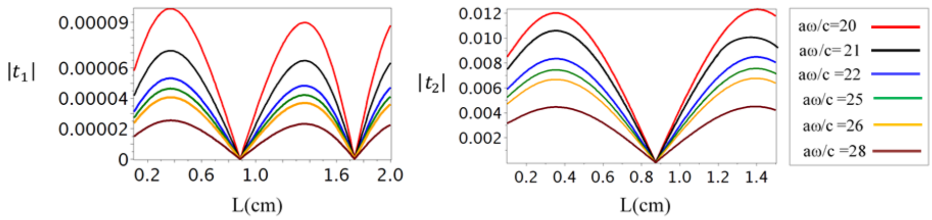

Figure 2: The magnitude of the transmission coefficients of the dominate modes ($n' = 1$) and first produced mode ($n' = 2$) of the final transmitted wave versus the length of the plasma valve and the incident wave frequency.

3. Numerical Results and Discussion

In this section, the computer simulation results of the transmission coefficients of the output wave from the plasma valve ($t_{n'}$) are presented and discussed. The numerical computations have been done by Maple software of version 10. The input data are as follows:

$a = 1\text{cm}$, $b = 0.02a$, $\varepsilon_{d1} = 2.4\varepsilon_0$, $\varepsilon_{d2} = 9\varepsilon_0$, $\omega_p = 6 \times 10^{11} Hz$, $j = m = n' = m' = 1$.

Figure 2, shows the magnitude of the transmission coefficients of the dominate modes ($n' = 1$) and first produced mode ($n' = 2$) of the final transmitted wave from the plasma versus the length of the plasma valve. The variations of the transmission coefficients of all modes in terms of the incident frequency wave are shown by the colored graphs in the mentioned diagrams.

As can be seen, by increasing the incident wave frequency, the magnitude of the transmission coefficients of both modes are decreased. Physically, this plasma valve behavior can be explained and interpreted based on the theory of permissible modes in Fabry-Prote resonators. As we can see in Figure 2, by changing the length of the plasma valve (open plasma resonator), the transmission coefficient of all modes reach their minimum values at certain lengths. It should be noted that, in this case, we are not dealing with a completely closed plasma resonator. As we know, in ideal electromagnetic resonators which are considered in a completely closed volume, the resonance frequencies are a function of all the geometric dimensions of the resonator in all degrees of freedom. However, in this paper it should be noted that the plasma valve is connected to the semi-bound waveguides in two ways and the volume of the plasma resonator is an open volume. Since the plasma rod is assumed to be almost non-washable, the minimum points for the coefficients of passage in each mode in Figure 2 are not dependent on the length of the plasma valve.

Figure 3, shows the magnitude of the transmission coefficients of the dominate modes ($n' = 1$) and first produced mode ($n' = 2$) of the final transmitted wave

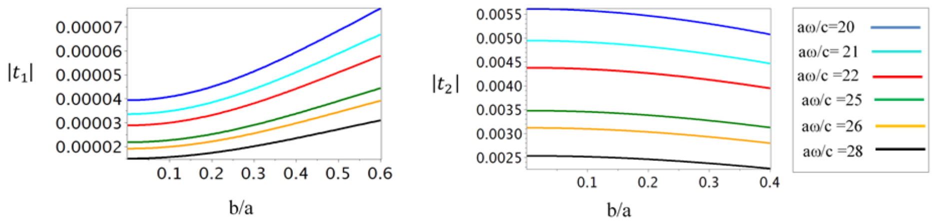

Figure 3: The magnitude of the transmission coefficients of the dominate modes ($n' = 1$) and first produced mode ($n' = 2$) of the output wave from the plasma, the radius of the plasma valve and the incident wave frequency.

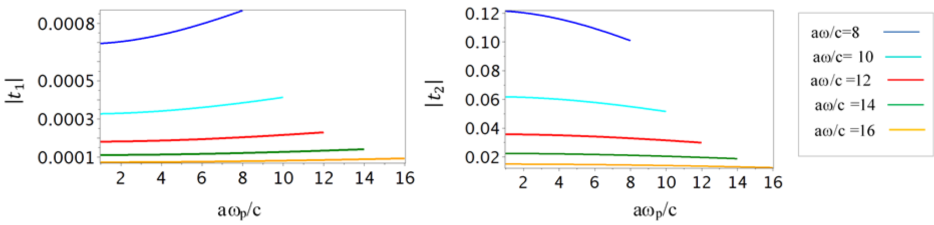

Figure 4: The magnitude of transmission coefficients of dominate modes ($n' = 1$) and first produced mode ($n' = 2$) of the final transmitted wave versus the plasma frequency and the incident wave frequency.

versus the radius of the plasma rod. The characteristic of colored graphs is the same as the Figure 2. In the mentioned diagrams, the length of the plasma valve is considered $L = 0.4cm$. As it can be shown, by increasing the radius of the plasma rod, the magnitude of the transmission coefficient of the dominate mode is decreased and the magnitude of the transmission coefficient of the produced mode is increased. Also, these graphs confirm decreasing the magnitude of transmission coefficients of all modes with increasing of the incident wave frequency.

Figure 4 shows the effect of plasma electric carrier density on transmission coefficients. The graphs of magnitude of variations of transmission coefficients for dominate modes ($n' = 1$) and the first produced mode ($n' = 2$) in final transmitted wave versus the variation of plasma frequency are presented in Figure 4. The colored graphs in this figure are also the same as the Figure 2. It shows that by increasing the plasma frequency, the magnitude of the transmission coefficient of the dominate mode is increased and the magnitude of the transmission coefficient of the produced mode is decreased.

4. Summary and Conclusion

In this work, the presence of a plasma valve in the path of an infinite waveguide with a cylindrical cross section was discussed. The plasma valve consisted of a plasma rod located in the center of a metal-walled dielectric waveguide. The calculations of transmission and reflection coefficients were performed on the assumption that the plasma rod is in the non-collision cold approximation. By injecting a downward wave from the left side of the plasma valve, and solving the equations of electromagnetic fields in symmetric modes of type TM, it was observed that the presence of system boundaries and the interaction of waves with them can create new modes with new quantities for passing and reflecting waves. The general vector potential solution was presented in different areas and after applying the appropriate boundary conditions, the pass and reflection coefficients for the final reflective waves and the final passing waves were obtained. It was shown that the mentioned coefficients were function of geometry dimensions, incident wave frequency and density of plasma valve. The graphs of the magnitude of the transmission coefficient of the final transmitted wave versus the mentioned quantities have been investigated. It was shown that at a constant frequency for the incident wave, the variations in the transmittance of the final waves in terms of variations in the length of the plasma valve are periodic and in some value of plasma valve length have a minimum value and in some value of plasma valve length have a maximum value.

Appendix

In this section, the law of conservation of energy in this configuration are investigated. As we know, the superposition rule for physical quantities is valid when the governing differential equations of them mentioned quantity, to be linear. This means that the linear combination of possible solutions of differential equation is also the solution of the differential equation. Since each of the electric field (E), the magnetic field(B) and the vector potential of an electromagnetic wave will be satisfied in the linear Maxwell's equations, therefore, in each point of space, the electric fields of waves in different modes can be combined with each other, and so for the magnetic field and vector potentials of the waves in different modes can be combined with each other [3, 18]. For a parameter such as the poynting vector which is derived from the product of two vectors H and E, it is obvious that the governing differential equation of the poynting vector is not linear because, it is obtained from the product of the two vectors H and E. In some cases, that the electromagnetic waves are propagated in different modes simultaneously, the total electric field and the total magnetic field can be obtained with the superposition rule in each point and the poynting vector will be introduced by the product of total vector H and total vector E in that point. This is also valid in configurations that include bouncing and return waves, which is fully cited in each of the following references to address the issue of energy survival theorem. Therefore, it should be noted that in our manuscript the relationship of energy conservation is established on $z = 0$ and $z = d$ and the energy conservation theorem is described by the following:

$$\Big[\Big(\sum_{n=1}^{\infty}\vec{E}_n\Big)\times\Big(\sum_{n=1}^{\infty}(\vec{H}_n)\Big)\Big]_{z=0} = \Big[\Big(\sum_{m=1}^{\infty}\vec{E}_m\Big)\times\Big(\sum_{m=1}^{\infty}\vec{H}_m\Big)\Big]_{z=0} + \Big[\vec{E}_j\times\vec{H}_j\Big]_{z=0}\Big[\Big(\sum_{n=1}^{\infty}\vec{E}_n\Big)\times\Big(\sum_{n=1}^{\infty}(\vec{H}_n)\Big)\Big]_{z=d} + \Big[\Big(\sum_{m'=1}^{\infty}\vec{E}_{m'}\Big)\times\Big(\sum_{m'=1}^{\infty}\vec{H}_{m'}\Big)\Big]_{z=d} = \Big[\Big(\sum_{n'=1}^{\infty}\vec{E}_{n'}\Big)\times\Big(\sum_{n'=1}^{\infty}\vec{H}_{n'}\Big)\Big]_{z=d}.$$

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this article.

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