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Collocation Method Based on Rational Gegenbauer Functions for Solving the Two Dimensional Stagnation Point Flow

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Abstract

This work applies rational Gegenbauer functions and a collocation scheme to solve the governing equation for two-dimensional fluid flow near a stagnation point, known as Hiemenz flow. We utilize a truncated series expansion of rational Gegenbauer functions on the semi-infinite interval and Gegenbauer–Gauss points to reduce the problem to a set of nonlinear algebraic equations. Newton's iteration technique is employed to solve these algebraic equations. The scheme is straightforward to implement, and our new results are compared with established numerical results, demonstrating the method's effectiveness and accuracy.

Keywords: Rational Gegenbauer functions, Collocation method, Stagnation point, Hiemenz flow, Boundary value problem.

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1. Introduction

Hiemenz flow refers to a specific type of steady, two-dimensional flow of a viscous incompressible fluid over a flat plate. This phenomenon is characterized by the

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development of a boundary layer along the plate due to fluid viscosity. The flow is named Hiemenz in honor of the German physicist Hermann Hiemenz who studied this phenomenon [1]. This flow has gained significant research interest due to its industrial and technological applications, including cooling electronic components, gas turbine blades, drying papers and films, tempering glass and metal during processing, and surface painting.

Similar to other flows and equations discussed in [2–7], Hiemenz stagnation point flows are governed by non-linear high-order equations. Finding numerical solutions is crucial since analytical solutions are often not explicitly obtainable. The governing equations are derived from the Navier-Stokes equations under specific assumptions [8], focusing on steady two-dimensional flow with an incompressible fluid and no pressure gradient in the flow direction. The governing Navier-Stokes equations simplify to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

(continuity equation) and

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

(momentum equation), where u is the velocity in the x-direction, v is the velocity in the y-direction, and p, ρ, ν are the fluid pressure, density and kinematic viscosity respectively. The boundary conditions are
1. At the plate ($y = 0$): $u = 0$ (constant velocity of the plate), $v = 0$,
2. Far from the plate ($y \to \infty$): $u = ax$,
where a being a constant. By introducing similarity transformations in the form

$$u = axf'(\xi), \quad v = -\sqrt{a\nu}f(\xi), \quad \xi = \sqrt{\frac{a}{\nu}}y,$$

the momentum equation (2) reduced to a nonlinear ordinary differential equation as the following [9]:

$$\frac{d^3 f}{d\xi^3} + f\frac{d^2 f}{d\xi^2} - (\frac{df}{d\xi})^2 + 1 = 0, \tag{3}$$

with transformed boundary conditions

$$f(0) = 0,\ f'(0) = 0,\ \lim_{\xi\to+\infty} f'(\xi) = 1. \tag{4}$$

In Hiemenz flow, shear stress τ is caused by fluid viscosity and is defined as the force acting parallel to the surface per unit area. In this context, it can be expressed as follows:

$$\tau = \mu\frac{\partial u}{\partial y}|_{y=0}, \tag{5}$$

where μ is the dynamic viscosity of the fluid and $\frac{\partial u}{\partial y}$ is the velocity gradient perpendicular to the flow direction. By placing similarity transformations in Equation (2), we have:

$$\tau = \mu a \sqrt{\frac{a}{\nu}} x f''(0).$$

Therefore, $f''(0)$ corresponds to surface shear stress and due to its relationship with physical values, we calculate $f''(0)$ in the results.
Howarth in [10] gave the numerical solution of axisymmetric stagnation point flow in three-dimensional with the finite difference method. Also, Gorla in [11] analyzed the dynamic properties of unstable fluid from an axisymmetric stagnation flow in a circular cylinder that created a harmonic motion during its flight. Recently, the authors in [12] solved the two-dimensional flow of fluid near a stagnation point by using radial basis functions (RBF). To solve this flow, Golbabai and Samadpour [9] used the rational Chebyshev collocation (RCC) method.

The primary aim of this work is to enhance and develop the rational Gegenbauer functions for the numerical solution of Heimenz flow, providing improved solutions compared to existing methods, which is a new and innovative work. The rational Gegenbauer approximation is flexible and efficiently approximates a wide range of functions, offering fast convergence for specific problems. This approximation increases the stability of calculations, especially for ill-conditioned problems. The orthogonality property of rational Gogenbauer functions is used to simplify calculations and expansions, while the compatibility property of these functions makes it adjust for boundary conditions with special domains. The features make it useful in various applications including numerical analysis and solving differential equations.

Guo [13, 14] introduced the Gegenbauer approximation to transform the main problem in an unlimited domain to a problem in a limited domain with the help of a mapping, and then used Gegenbauer polynomials to get the numerical solution of the problem. The plan of this scheme involves reducing the problem to a set of algebraic equations with the expansion of $f(\xi)$ in parts of rational Gegenbauer functions with unknown coefficients. Rational Gegenbauer functions are considered a complete spectral basis for semi-infinite intervals [15–18]. The authors of [19] applied the collocation method based on these functions to solve the laminar boundary layer equation. Additionally, Parand et al. [20] applied rational Gegenbauer functions with the Quasi-linearization method for boundary layer flow involving Powell–Eyring non-Newtonian fluid.

The remainder of this article is structured as follows: Section 2 details the implementation of rational Gegenbauer functions. In Section 3, we use these functions and the collocation scheme to solve our studied model. Also, in this section, the convergence rate of the approximation of rational Gegenbauer functions is given. Results and discussions of the proposed scheme are shown in Section 4. Finally, Section 5 is dedicated to a conclusion.

2. Rational Gegenbauer interpolation

In this section, we will begin by defining the Gegenbauer polynomials of degree m as follows:

$$G_m^\alpha(\xi)=\sum_{i=0}^{[\frac{m}{2}]}(-1)^i\frac{\Gamma(\xi+\alpha-i)}{i!(m-2i)!\Gamma(\alpha)}(2\xi)^{m-2i},\quad \alpha>-\frac{1}{2}.$$

Here, $m=0,1,2,\ldots$ and Γ denotes the Gamma function. These polynomials are orthogonal over the interval $[-1.1]$ with the weight function $w(\xi)=(1-\xi^2)^{-\frac{1}{2}}$. For Gegenbauer polynomials, α can vary in the interval $(-\frac{1}{2},+\infty)$; specifically, it corresponds to the Chebyshev polynomials of the first kind $\alpha=0$, the second kind of Chebyshev polynomials $\alpha=1$, and Legendre polynomials $\alpha=0.5$. Now, we define rational Gegenbauer functions $RG_m^\alpha(\xi)$, for scaling/stretching factor $L>0$, by [13]:

$$RG_m^\alpha(\xi)=G_m^\alpha\left(\frac{\xi-L}{\xi+L}\right),\ m=0,1,2,\ldots, \tag{6}$$

where $G_m^\alpha(\xi)$ is a Gegenbauer polynomial of degree m and order α. The transformation $\frac{\xi-L}{\xi+L}$ in (6) is selected which the interval $[-1,1]$ is converted to $J=[0,\infty)$. The optimal value of the Boyd offering for the map parameter L can be seen in [21, 22].

The rational Gegenbauer function $RG_m^\alpha(\xi)$ is an eigenfunction for the following Sturm-Liouville problem:

$$(\xi+L)\frac{\sqrt{\xi}}{L}\frac{d}{d\xi}[(\xi+L)\sqrt{\xi}\frac{d}{d\xi}RG_m^\alpha(\xi)]+\alpha\left(\frac{\xi^2-L^2}{L}\right)\frac{d}{d\xi}RG_m^\alpha(\xi)+m(m+2\alpha)RG_m^\alpha(\xi)=0.$$

Additionally, these functions can be defined using the following recurrence relation:

$$RG_0^\alpha(\xi)=1,\quad RG_1^\alpha(\xi)=2\alpha\frac{\xi-L}{\xi+L},$$
$$RG_{m+1}^\alpha(\xi)=\frac{1}{m+1}\left[2\left(\frac{\xi-L}{\xi+L}\right)(m+\alpha)RG_m^\alpha(\xi)-(m+2\alpha-1)RG_{m-1}^\alpha(\xi)\right],\quad m>1.$$

Now, let us define

$$L^2_\omega(J)=\{f:J\to\mathbb{R}\text{ such that }f\text{ is measurable function and }||f||_\omega<\infty\},$$

for the weight function $\omega_\alpha(\xi)=\frac{2L}{(\xi+L)^2}[1-(\frac{\xi-L}{\xi+L})]^{\alpha-\frac{1}{2}}$ which is an integrable, non-negative, real-valued function on J and

$$||f||_\omega=\left(\int_0^\infty|f(\xi)|^2\omega_\alpha(\xi)d\xi\right)^{\frac{1}{2}},$$

is the norm determined by the scalar product

$$< f(\xi), h(\xi) >_\omega = \int_0^\infty f(\xi)h(\xi)\omega_\alpha(\xi)d\xi. \tag{7}$$

Therefore, $\{RG_m^\alpha(\xi)\}_{m\geq 0}$ expresses one set that is orthogonal under Equation (7),

$$< RG_m^\alpha(\xi), RG_n^\alpha(\xi) >_\omega = \frac{\pi 2^{1-2\alpha}\Gamma(n+2\alpha)}{n!(n+\alpha)(\Gamma(\alpha))^2}\delta_{mn},$$

where δ_{mn} is the Kronecker delta function. The current system is completed in $L^2_\omega(J)$. Therefore, for each function $f \in L^2_\omega(J)$, we have the following series expansion:

$$f(\xi) = \sum_{j=0}^{+\infty} a_j RG_j^\alpha(\xi),$$

with

$$a_j = \frac{< f(\xi), RG_j^\alpha(\xi) >_\omega}{||RG_j^\alpha(\xi)||^2_\omega}.$$

3. Solving the stagnation point flow

Let $\mathbf{RG}_M^\alpha = \text{span}\{RG_0^\alpha, RG_1^\alpha, ..., RG_M^\alpha\}$ for any positive integer M. Generally, the $L^2_\omega(J)$- orthogonal projection $I_M : L^2_\omega(J) \to \mathbf{RG}_M^\alpha$ for each $f \in L^2_\omega(J)$ would be defined as below:

$$< I_M f - f, RG_i^\alpha >_\omega = 0, \quad \forall\ RG_i^\alpha \in \mathbf{RG}_M^\alpha.$$

Equivalently,

$$I_M f(\xi) = \sum_{j=0}^{M} a_j RG_j^\alpha(\xi). \tag{8}$$

In this paper, we choose $L \simeq \sqrt[4]{\frac{M}{2}}$ according to Weideman's theory [23]. Note that form the definitions of $RG_j^\alpha(\xi)$ and $I_M f(\xi)$, we have $\frac{d}{d\xi}RG_j^\alpha(\infty) = 0, j = 0, 1, ..., M$ and $I_M f'(\infty) = 0$. To validate the boundary condition (4), a simple part is added to Equation (4) so that we get the following form:

$$I_M f(\xi) = \xi + \sum_{j=0}^{M} a_j RG_j^\alpha(\xi), \tag{9}$$

where $I_M f'(\infty) = 1$. It can be seen that the boundary condition $f'(\infty) = 1$ is completely satisfied. Now, to solve problem (3)-(4), we first set to (9), $I_M(f)$

approximately from f and find the unknown coefficients $a_j,\ j=0,1,...,M$. The residual function of (3) is as follows:

$$Res_M(\xi)=\frac{d^3I_Mf}{d\xi^3}+I_Mf\ \frac{d^2I_Mf}{d\xi^2}-\left(\frac{dI_Mf}{d\xi}\right)^2+1.$$

So from the collocation method, we have

$$Res_M(\xi)|_{\xi=\xi_i}=0,\ i=1,...,M-1, \tag{10}$$

$$I_Mf(0)=0, \quad I_Mf'(0)=0, \tag{11}$$

where $\xi_i,(i=1,2,...,M-1)$ are the rational Gegenbauer-Gauss points. It is interesting to note that, ξ_i's are zeros of the polynomial $RG^{\alpha}_{M+1}(\xi)+RG^{\alpha}_{M}(\xi)$. Equations (10) and (11) creates $M+1$ nonlinear algebraic equations which can be used to obtain the numerical approximation solutions via Newton's method.

3.1 Convergence and error estimations

According to [24], in order to estimate $||f(\xi)-I_Mf(\xi)||^2_{\omega}$, we use Equation (8) and write the error as:

$$\begin{aligned}
||e_M||^2_{\omega} &= ||f(\xi)-I_Mf(\xi)||^2_{\omega}=||\sum_{j=M}^{+\infty}a_jRG^{\alpha}_j(\xi)||^2_{\omega}\\
&= \sum_{m=M}^{+\infty}\sum_{n=M}^{+\infty}a_ma_n<RG^{\alpha}_m(\xi),RG^{\alpha}_n(\xi)>_{\omega}\\
&= \sum_{m=M}^{+\infty}\sum_{n=M}^{+\infty}a_ma_n\frac{\pi 2^{1-2\alpha}\Gamma(n+2\alpha)}{n!(n+\alpha)(\Gamma(\alpha))^2}\delta_{mn}\\
&= \sum_{m=M}^{+\infty}\frac{\pi 2^{1-2\alpha}\Gamma(m+2\alpha)a_m^2}{m!(m+\alpha)(\Gamma(\alpha))^2}\\
&= \sum_{m=M}^{+\infty}\frac{\pi 2^{1-2\alpha}\Gamma(m+2\alpha)}{m!(m+\alpha)(\Gamma(\alpha))^2}\left(\frac{m!(m+\alpha)(\Gamma(\alpha))^2}{\pi 2^{1-2\alpha}\Gamma(m+2\alpha)}\right)^2<f(\xi),RG^{\alpha}_m(\xi)>^2_{\omega}\\
&= \sum_{m=M}^{+\infty}\frac{m!(m+\alpha)(\Gamma(\alpha))^2}{\pi 2^{1-2\alpha}\Gamma(m+2\alpha)}<f(\xi),RG^{\alpha}_m(\xi)>^2_{\omega}.
\end{aligned}$$

The last relation obtained is shown that the convergence rate is included function $f(\xi)$. Also, the following theorem presents an upper bound for the error, as detailed in [24].

Theorem 3.1. *For any* $\eta=\frac{\xi-L}{\xi+L}$, $F(\eta)=f(\Phi(\eta))$ *on* $[-1,1]$ *and* $M_i=\max|F^{(i)}(\eta)|$, *we have:*

$$||e_M||^2_{\omega}\leq\sum_{m=M}^{+\infty}\frac{\pi\ 2^{1-2m-2\alpha}\Gamma(m+2\alpha)M_m^2}{(m+\alpha)\Gamma^2(m+\alpha)\ \Gamma(m+1)}.$$

Table 1: Comparison of $||Res||^2$ in implementing the rational Gegenbauer method for Hiemenz flow with different α.

α	$M = 10$	$M = 20$	$M = 30$	$M = 40$
-0.48	$2.62418E-04$	$7.40814E-06$	$6.47648E-09$	$3.80778E-12$
-0.01	$2.65025E-04$	$2.54409E-05$	$3.79708E-08$	$3.35746E-11$
0.5	$6.63230E-04$	$7.86611E-05$	$2.05367E-07$	$2.74548E-10$
1	$3.75565E-03$	$7.35231E-07$	$1.52807E-07$	$7.62892E-12$
1.5	$5.03195E-03$	$4.07776E-04$	$2.98237E-06$	$8.25476E-09$
2	$1.05290E-02$	$7.38051E-04$	$8.76501E-06$	$3.38341E-08$
2.5	$1.92001E-02$	$1.19115E-03$	$2.23594E-05$	$1.18322E-07$
3	$3.15689E-02$	$1.78592E-03$	$5.05122E-05$	$3.60312E-07$

4. Results and discussion

This section is devoted to the presentation of some numerical solutions obtained by applying the rational Gegenbauer functions method. We use $\alpha = -0.48, -0.01, 0.5, 1, 1.5, 2, 2.5$ and 3 which leads to $f''(0) = 1.2325876568$. To make a comparison in Table 1, we use the measure $||Res||^2$ with form:

$$||Res||^2 = \int_0^\infty Res_M^2(\xi)d\xi,$$

and it can be seen that increasing in number of Gauss points causes the convergence of this method. Considering this table, by increasing the amount of M in the proposed method, the amount of error is significantly reduced. To select the appropriate value from Table 1, the values are $||Res||^2$ with $M = 40$ for different amounts of α in Figure 1. This graph shows the limits that we can be used to obtain executable results. We have the lowest error rate at $\alpha = -0.48$ in this figure and therefore we prefer the value $\alpha = -0.48$ for the stagnation point flow. In Table 2 and Figure 2, the coefficients a_i and the logarithmic figure of the absolute coefficients $|a_i|$ of the approximate solution to the problem is shown by the Gegenbauer functions by selecting $M = 40$ and $\alpha = -0.48$. The stability and convergence of the rational Gegenbauer collocation method are shown by this graph and table. In addition, we calculate the convergence exponential index called r, which is used in Figure 3 with

$$r = \lim_{j\to\infty} \frac{\log|\log(|a_j|)|}{\log(j)}.$$

Figure 3 shows that it is approximately $r < 1$ and corresponds to Boyd [22], so we infer the subgeometric convergence of the spectral approximation (9).

Approximate solutions of $f(\xi)$ are displayed in Table 3 with the proposed method for $M = 40$ and the resulting solutions, has been compared with solutions

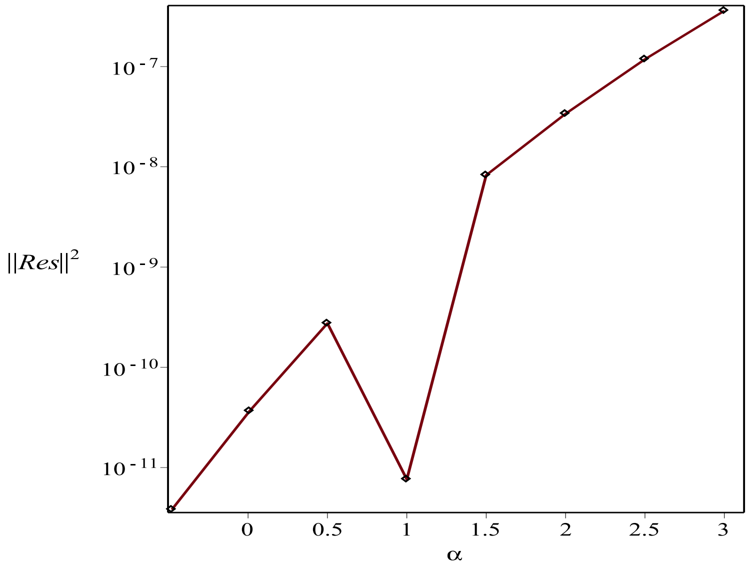

Figure 1: Graph for $||Res||^2$ of presented scheme with $M = 40$ and different numbers of α.

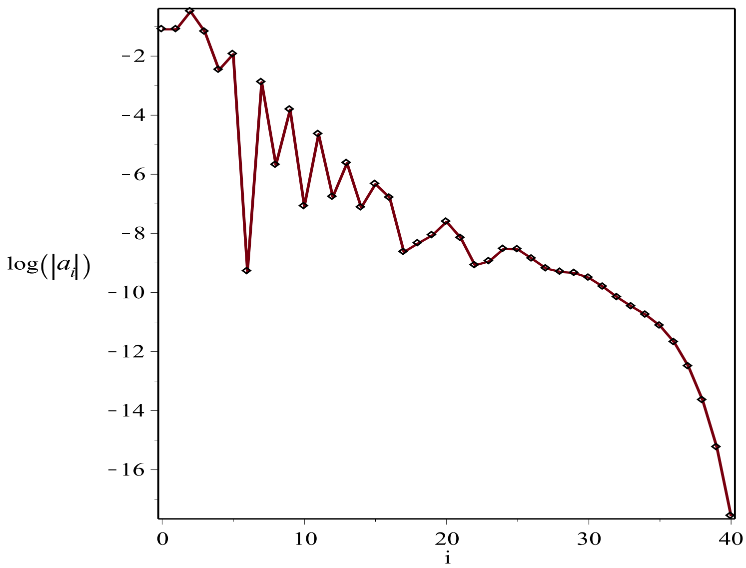

Figure 2: Logarithmic figure for absolute coefficients $|a_i|$ when $\alpha = -0.48$ and $M = 40$ for the rational Gegenbauer functions.

Table 2: Approximations of coefficients a_i of the present method for $M = 40$.

i	a_i	i	a_i	i	a_i		
0	$-3.35471E-01$	11	$9.64783E-03$	22	$1.13788E-04$	33	$2.83090E-05$
1	$3.35583E-01$	12	$1.15197E-03$	23	$1.30442E-04$	34	$2.13539E-05$
2	$-6.14051E-01$	13	$-3.61675E-03$	24	$1.97526E-04$	35	$1.47523E-05$
3	$3.09584E-01$	14	$-8.05463E-04$	25	$1.95448E-04$	36	$8.43683E-06$
4	$8.48399E-02$	15	$1.77945E-03$	26	$1.42810E-04$	37	$3.72643E-06$
5	$-1.44513E-01$	16	$1.12206E-03$	27	$1.02288E-04$	38	$1.18421E-06$
6	$9.29174E-05$	17	$-1.78145E-04$	28	$9.11053E-05$	39	$2.40745E-07$
7	$5.59343E-02$	18	$-2.39270E-04$	29	$8.76080E-05$	40	$2.35017E-08$
8	$-3.40220E-03$	19	$3.14593E-04$	30	$7.46637E-05$		
9	$-2.21294E-02$	20	$4.96876E-04$	31	$5.51198E-05$		
10	$8.39598E-04$	21	$2.86826E-04$	32	$3.85812E-05$		

of Howarth [10], Abbasbandy [12], Golbabai [9] and the Runge-Kutta method of four order. Also, in this table, in the last column, the following rational Chebyshev-Gauss-Radau collocation points expressed in [9],

$$\tau_i = L\frac{1+x_i}{1-x_i}, \quad x_i = -cos\left(\frac{2i\pi}{2M+1}\right), \quad i = 0, 1, ..., M,$$

combined with the rational Gegenbauer method (RGM2) is used which is similar to the results obtained with the results obtained with Gauss-Gegenbauer collocation points in the rational Gegenbauer method stated in this article (RGM1). In addition, the graphs $f(\xi), f'(\xi)$ and $f''(\xi)$ are shown in Figure 4 for $M = 40$ and it can be seen that the graphs of this figure are in good agreement with results of [9, 10, 12].

$f''(\xi)$ at the zero point is very important for the stagnation point flow. Comparing $f''(0)$ obtained by the present scheme with the results calculated in [9, 10, 12] and Runge-Kutta scheme of fourth order in Table 3, we find that the proposed method is very accurate.

5. Conclusion

In this study, we used the rational Gegenbauer method to approximate the solution of the third-order nonlinear differential equation related to the stagnation point flow. This approach allowed us to transform the flow equation into a set of algebraic equations. The numerical solution obtained using the rational Gegenbauer scheme was consistent with the approximations presented in [9, 10, 12] and showed the convergence and strong effectiveness of the method. In the context

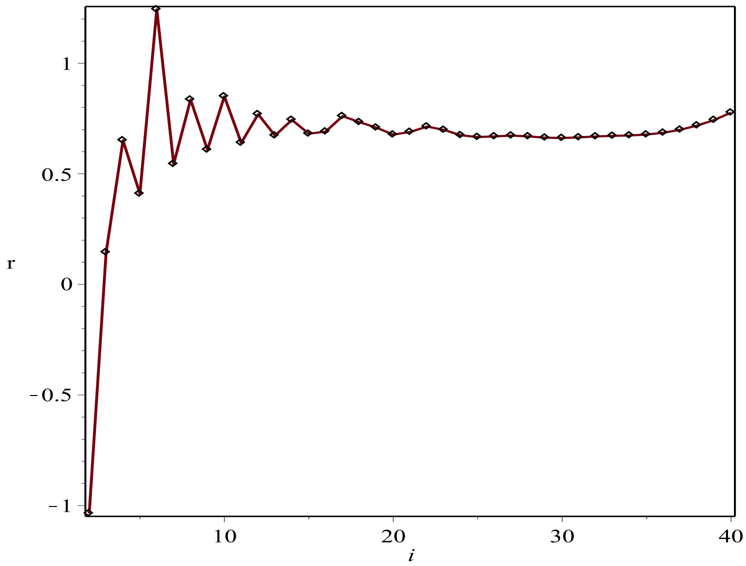

Figure 3: Graph of convergence index r for different numbers of i.

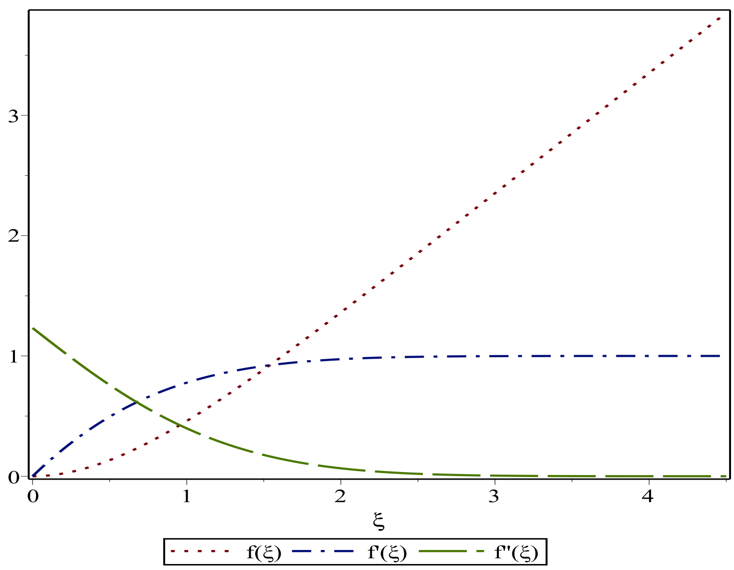

Figure 4: Approximations $f(\xi), f'(\xi)$ and $f''(\xi)$ for the Hiemenz flow for $M = 40$.

Table 3: Comparing the results of $f(\xi)$ and $f''(0)$ from the present scheme using rational Gauss-Gegenbauer collocation points (RGM1) and rational Chebyshev-Gauss-Radau collocation points (RGM2) with methods from [9, 10, 12] and the Runge-Kutta method.

ξ	Method of [10]	Method of [12]	Method of [9]	Runge-Kutta	RGM1	RGM2
0.0	0	0	0	0	0	0
0.2	0.0232	0.233355	0.0233222570	0.0233222492	0.0233222570	0.0233222570
0.6	0.1867	0.186715	0.1867009935	0.1867009886	0.1867009935	0.1867009935
1.0	0.4592	0.459236	0.4592270171	0.4592270144	0.4592270170	0.4592270170
1.4	0.7966	0.796657	0.7966517836	0.7966517822	0.7966517835	0.7966517835
1.8	1.1688	1.168855	1.1688554755	1.1688554750	1.1688554755	1.1688554755
2.0	1.3619	1.361968	1.3619741619	1.3619741617	1.3619741619	1.3619741618
2.4	1.7552	1.755238	1.7552538766	1.7552538771	1.7552538764	1.7552538766
2.8	2.1529	2.152965	2.1529965067	2.1529965081	2.1529965069	2.1529965067
3	2.3525	2.352516	2.3525566747	2.3525566765	2.3525566747	2.3525566746
$f''(0)$	1.2326	1.229742	1.2325876434	1.2325876312	1.2325876568	1.2325876568

of Hiemenz flow, we specifically calculated $f''(0)$ which is very important for the analysis and compared the results with existing calculations. This highlighted the reliability of rational Gegenbauer method for dealing with such problems.

Conflicts of Interest. The author declares that she has no conflicts of interest regarding the publication of this article.

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