Mathematics Interdisciplinary Research **11** (2) (2026) 113 – 124

Original Scientific Paper

Stability of 2-Domination Number of a Graph

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Abstract

This paper delves into the stability of the 2-domination number in simple undirected graphs. The 2-domination number of a graph G, $\gamma_2(G)$, represents the minimum size of a vertex subset where every other vertex in the graph is adjacent to at least two members of the subset. We define the 2-domination stability, $st_{\gamma_2}(G)$, as the smallest number of vertices whose removal causes a change in $\gamma_2(G)$. Our primary contributions include computing this parameter for specific graphs, establishing various bounds for this stability, and determining its behavior under certain graph operations combining two graphs.

Keywords: Dominating set, 2-Domination number, Stability, Operation.

2010 Mathematics Subject Classification: 05C05; 05C69.

How to cite this article
M. Mehrabam and S. Alikhani, Stability of 2-domination number of a graph, *Math. Interdisc. Res.* **11** (2) (2026) 113-124.

1. Introduction

Let $G = (V, E)$ be a simple graph with finite number of vertices. The open neighborhood of a vertex $v \in V(G)$ is the set of vertices that are adjacent to v, but not including v itself, $N(v) = \{u \in V(G) : uv \in E(G)\}$ and the closed neighborhood of a vertex $v \in V(G)$ is the open neighborhood of v along with the vertex v itself and is denoted as $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V(G)$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. The private neighborhood $pn(v, S)$ of $v \in S$ is defined by $pn(v, S) = N(v) - N(S - \{v\})$, equivalently, $pn(v, S) = \{u \in V | N(u) \cap S = \{v\}\}$.

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Academic Editor: Gholam Hossein Fath-Tabar
Received 13 September 2025, Accepted 23 November 2025
DOI: 10.22052/MIR.2025.257573.1540

The degree of a vertex v denoted as $deg(v)$, is the number of edges incident to that vertex, which is equal to $|N(v)|$. A leaf of a tree is a vertex of degree 1. A subset $D \subseteq V(G)$ is called a dominating set of G if every vertex $v \in V \setminus D$ has at least one neighbor in D; that is, $N(v) \cap D \neq \emptyset \quad$ for all $v \in V \setminus D$. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G (see [1, 2]). A domination-critical vertex in a graph G is a vertex whose removal decreases the domination number. It is easy to observe that for any graph G, we have $\gamma(G) - 1 \leq \gamma(G + e) \leq \gamma(G)$ for every edge $e \notin E(G)$. Sumner and Blitch in [3] have defined domination critical graphs. A graph G is said to be domination critical, or γ-critical, if $\gamma(G + e) = \gamma(G) - 1$ for every edge e in the complement G^c of G. A graph is said to be domination stable, or γ-stable, if $\gamma(G) = \gamma(G + e)$ for every edge e in the complement G^c of G. For detailed information and results regarding the concept of domination critical graphs, we refer interested readers to the papers [3–5]. Bauer et al. introduced the concept of domination stability in graphs in 1983 [6]. After then, it was studied by Rad, Sharifi and Krzywkowski in [7]. Stability for different types of domination parameters has been investigated in the literature, for example, in [8–12]. This subject has been considered and studied for other graph parameters. For example, see [13–15].

A subset $D \subseteq V(G)$ is called a 2-dominating set of the graph G if every vertex $v \in V \setminus D$ has at least two neighbors in D; that is, $|N(v) \cap D| \geq 2 \quad$ for all $v \in V \setminus D$. The 2-domination number of G, denoted by $\gamma_2(G)$, is the minimum cardinality of a 2-dominating set in G: $\gamma_2(G) = \min\{|D| \,|\, D \subseteq V(G), \forall v \in V \setminus D, \, |N(v) \cap D| \geq 2\}$. For more details see [16].

In the next section, we define the stability of 2-domination number and compute the value of the stability of 2-domination number for some special classes of graphs. We find some bounds on the stability of 2-domination number in Section 3. The stability of 2-domination number of some operations of two graphs is studied in Section 4. Finally, we conclude the paper in Section 5.

2. Stability of 2-domination number of certain graphs

In this section, we first state the stability of 2-domination number and compute the stability of 2-domination number of the graph G for some specific graphs.

Definition 2.1. Let G be a graph of order at least 3. The stability of 2-domination number of G, denoted by $st_{\gamma_2}(G)$, is the minimum number of vertices that must be removed from G in order to change its 2-domination number.

2.1 Results for specific graphs

We start with the following observation:

Observation 2.2. ([16]). *If P_n, C_n and W_n are the path graph, the cycle graph and the wheel graph of order $n \geq 4$, then*

(i)

$$\gamma_2(P_n) = \begin{cases} \frac{n}{2}+1, & \text{if } n \text{ is even},\\ \frac{n-1}{2}+1, & \text{if } n \text{ is odd}. \end{cases}$$

(ii)

$$\gamma_2(C_n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even},\\ \frac{n+1}{2}, & \text{if } n \text{ is odd}. \end{cases}$$

(iii) For every $n \geq 3$,

$$\gamma_2(W_n) = \begin{cases} 2, & \text{if } n = 3,4,\\ \lfloor \frac{n+1}{3} \rfloor + 1, & \text{if } n \geq 5. \end{cases}$$

We obtain the stability of 2-domination number of some specific graphs.

Proposition 2.3. *(i) For $n \geq 4$, $st_{\gamma_2}(P_n) = 3$.*

(ii) $st_{\gamma_2}(C_4) = 3$ and for $n \geq 5$, $st_{\gamma_2}(C_n) = \begin{cases} 2, & \text{if } n \text{ is odd},\\ 3, & \text{if } n \text{ is even}. \end{cases}$

(iii) If W_n is a wheel graph (join of K_1 and C_{n-1}, i.e., $K_1 \vee C_{n-1}$), then for $n \geq 5$, $st_{\gamma_2}(W_n) = 2$.

Proof. (i) By removing the three first consecutive vertices of P_n, the 2-domination number of P_n will be changed but by removing two vertices, this parameter does not change. So we have the result.

(ii) Suppose that $V(C_n) = \{v_1, v_2, \dots, v_n\}$. For odd n, by removing two consecutive vertices $\{v_1, v_2\}$, we will have P_{n-2} which its 2-domination number is one less than the 2-domination number of C_n. For even n, we need to remove three consecutive vertices $\{v_1, v_2, v_3\}$.

(iii) Let $W_n = K_1 \vee C_{n-1}$. By removing two adjacent vertices of C_{n-1}, the 2-domination number of W_n, which is $\lfloor \frac{n+1}{3} \rfloor + 1$, will be changed.

□

Now we obtain the stability of 2-domination number of the friendship graph and the book graph. The friendship graph F_n is a graph that can be constructed by coalescing n copies of the cycle graph C_3 of length 3 with a common vertex. The friendship graph F_n is a graph with the property that every two vertices have exactly one neighbor in common are exactly the friendship graphs [17]. The n-book graph ($n \geq 2$) is defined as the Cartesian product $K_{1,n} \square P_2$. We call every C_4 in the book graph B_n, a page of B_n. All pages in B_n have a common side v_1v_2. Figure 1 shows the friendship graph F_n and the book graph B_n.

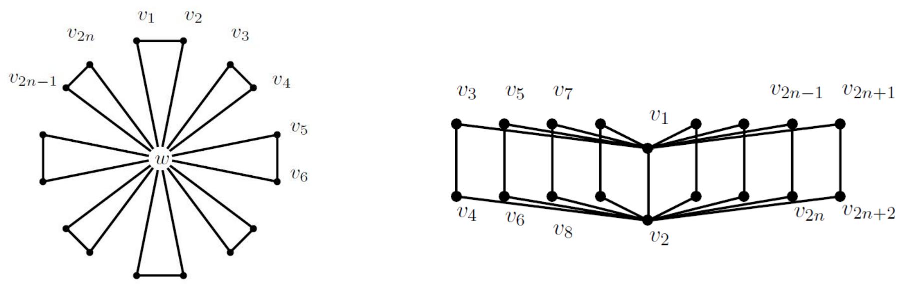

Figure 1: Friendship graph F_n and book graph B_n, respectively.

The following observation gives the 2-domination number of the friendship graph and the book graph.

Observation 2.4. ([16]).

(i) *For* $n \geq 1$, $\gamma_2(F_n) = n + 1$.

(ii) *For* $n \geq 2$, $\gamma_2(B_n) = n + 1$.

The following proposition gives the stability of 2-domination number of F_n and B_n.

Proposition 2.5. *(i)* *For any* $n \geq 2$, $st_{\gamma_2}(F_n) = 1$.

(ii) *For any* n, $st_{\gamma_2}(B_n) = 1$.

Proof. (i) By removing the central vertex (the vertex w in Figure 1), the 2-domination number change.

(ii) We have the result by removing two vertices v_1, v_2 of B_n (Figure 1).

□

Observation 2.6. ([16]).

(i) *If* K_n *is a complete graph for every* $n \geq 2$, $\gamma_2(K_n) = 2$.

(ii) *For* $n \geq 3$, $\gamma_2(K_{1,n}) = n - 1$.

(iii) *For every* $m, n \geq 4$, $\gamma_2(K_{m,n}) = 4$.

Now, we state the value of the stability of 2-domination number of K_n, $K_{1,n}$ and $K_{m,n}$, which are easy to obtain.

Proposition 2.7. *(i)* *For every* $n \geq 2$, $st_{\gamma_2}(K_n) = n - 1$.

(ii) *For* $n \geq 2$, $st_{\gamma_2}(K_{1,n}) = 1$.

(iii) $st_{\gamma_2}(K_{m,n}) = \begin{cases} 1, & \text{if } m = 1, \text{ or } n = 1, \\ 2, & \text{if } m, n \geq 2. \end{cases}$

2.2 Results for cactus graphs

In this subsection, we obtain the stability of 2-domination number of some of cactus graphs. A cactus graph is a connected graph where each edge belongs to at most one cycle. Therefore, every block of a cactus graph is either a single edge or a cycle. When all blocks in a cactus graph G are cycles of identical length m, the graph is called an m-uniform cactus.

A triangular cactus is a connected graph where each block is a triangle, making it a 3-uniform cactus. A vertex that belongs to multiple triangles is called a cut-vertex. When every triangle contains at most two cut-vertices, and each cut-vertex is shared by exactly two triangles, the graph forms a chain triangular cactus. The length of this chain is the number of triangles it contains. Such a structure, denoted by T_n, has $2n+1$ vertices and $3n$ edges ([18]) (see Figure 2). By extending this idea to cycles of length four, we define square cacti, where each block is a C_4. Chain square cacti, denoted by Q_n, vary depending on how internal squares connect to each other (see Figure 2). If the cut-vertices of a square are adjacent, it is called an ortho-square; otherwise, it is a para-square. The chain consisting solely of para-squares is denoted by Q_n, while the chain formed by ortho-squares is denoted by O_n (illustrated in Figure 2).

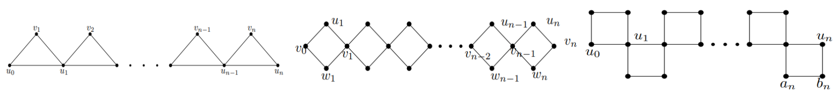

Figure 2: The cactus T_n, Q_n and O_n, respectively.

Observation 2.8. *(i)* $\gamma_2(T_n) = \gamma_2(Q_n) = \lceil \frac{n+2}{2} \rceil$.

(ii) *For the ortho-chain square cactus graph,* $\gamma_2(O_n) = n + 1$.

The following theorem gives the stability of 2-domination number of these three cactus, which is easy to obtain.

Proposition 2.9. *For* $n \geq 2$, $st_{\gamma_2}(T_n) = st_{\gamma_2}(Q_n) = st_{\gamma_2}(O_n) = 2$.

We close this section by the following theorem:

Theorem 2.10. *(i) There exist graphs* G *and* H *with the same* 2*-domination stability such that* $|\gamma_2(G) - \gamma_2(H)|$ *is arbitrarily large.*

(ii) There exist graphs G *and* H *with the same* 2*-domination number such that* $|st_{\gamma_2}(G) - st_{\gamma_2}(H)|$ *is arbitrarily large.*

Proof. (i) Suppose that G is the helm graph which is obtained from a wheel graph of order n (i.e., $W_n = K_1 \vee C_{n-1}$) by appending a single pendant edge to each vertex of cycle graph. We know that for $n \geq 3$, $\gamma_2(G) = \lfloor \frac{n}{2} \rfloor + 1$ (see [16]). If we remove the central vertex from G, say v, then

$$\gamma_2(G - \{v\}) = n \neq \lfloor \frac{n}{2} \rfloor + 1,$$

and so $st_{\gamma_2}(G) = 1$.

Now consider the graph H, which is obtained from a wheel graph of order n (i.e., $W_n = K_1 \vee C_{n-1}$) by inserting a new vertex between any pair of adjacent vertices on the cycle C_{n-1}. It is easy to see that for $n \geq 3$, $\gamma_2(H) = n$ (see [16]). By removing one vertex from C_{n-1}, say v, we have

$$\gamma_2(G - \{v\}) = n - 1 \neq n,$$

and so $st_{\gamma_2}(H) = 1$. So we have the result.

(ii) Consider the graph $G = P_4$ and $H = K_n$. We have $\gamma_2(P_4) = \gamma_2(K_n) = 2$. On the other hand, $st_{\gamma_2}(P_n) = 3$ and $st_{\gamma_2}(K_n) = n - 1$ and so we have the result.

□

3. Bounds on $st_{\gamma_2}(G)$

In this section, we derive some bounds related to the stability of 2-domination number in graphs. First, we study the relationship between the stabilities of 2-domination numbers of graphs G and $G - v$, where $v \in V(G)$. Also, we obtain upper bounds for st_{γ_2}(G).

Proposition 3.1. *Let G be a graph and v be a vertex of G. Then*

$$st_{\gamma_2}(G) \leq st_{\gamma_2}(G - v) + 1.$$

Proof. If $\gamma_2(G) = \gamma_2(G - v)$, then we have $st_{\gamma_2}(G) \leq st_{\gamma_2}(G - v) + 1$. If $\gamma_2(G) \neq \gamma_2(G - v)$, then removing v change the 2-domination number, which implies $st_{\gamma_2}(G) = 1$, and so the inequality holds trivially. □

By applying this proposition iteratively for vertices $v_1, v_2, \ldots, v_s$ with $1 \leq s \leq n - 2$ and $n = |V(G)|$, we get

$$st_{\gamma_2}(G) \leq st_{\gamma_2}(G - v_1 - \cdots - v_s) + s.$$

Using this formula, various upper bounds for $st_{\gamma_2}(G)$ can be obtained. In the next theorem, we state some of these upper bounds. The proof for each case involves removing vertices from G until the induced subgraph satisfying the hypothesis appears. Then, applying the above inequality and the known value of $st_{\gamma_2}(G - v_1 - \cdots - v_s)$ yields the result.

Theorem 3.2. *Let G be a simple graph of order $n \geq 2$. Then:*

(i) $st_{\gamma_2}(G) \leq n - 1$.

(ii) If G has the star graph $K_{1,t}$ as the induced subgraph with $t \geq 3$, then $st_{\gamma_2}(G) \leq n - t$.

We need the following theorem:

Theorem 3.3. ([19]).

(i) If the minimum degree $\delta(G)$ is 0 or 1, then $\gamma_2(G)$ can be equal to n.

(ii) If $\delta(G) = 2$, then $\gamma_2(G) \leq \frac{2}{3}n$.

(iii) If $\delta(G) \geq 3$, then $\gamma_2(G) \leq \frac{1}{2}n$.

Now we state and prove the following theorem.

Theorem 3.4. *If G is a graph of order n, then*

$$st_{\gamma_2}(G) \leq n + 1 - \gamma_2(G).$$

Proof. Let $st_{\gamma_2}(G) = k$. By definition of stability, removal of any set $S = \{v_1, \ldots, v_{k-1}\}$ with $k-1$ vertices preserves the 2-domination number, i.e., $\gamma_2(G) = \gamma_2(G - v_1) = \cdots = \gamma_2(G - v_1 - \cdots - v_{k-1})$. For the remaining graph $G - S$ of order $n - (k-1) = n - k + 1$, we use the known upper bound for 2-domination in part (i) of Theorem 3.3, $\gamma_2(G - S) \leq n - k + 1$. Since $\gamma_2(G - S) = \gamma_2(G)$, we have $\gamma_2(G) \leq n - k + 1$. Solving for k yields $k \leq n + 1 - \gamma_2(G)$. Therefore, we have the result. □

Corollary 3.5. *Let G be a graph of order $n \geq 2$. If $st_{\gamma_2}(G) = n - k$, then $\gamma_2(G) \leq k + 1$.*

Proof. It is a direct consequence of Theorem 3.4. □

Volkmann in [20] presented the Nordhaus-Gaddum-type result for the 2-domination number as follows:

Theorem 3.6. ([20]). *If G is a graph of order n and $\overline{G}$ is the complement of G, then*

$$\gamma_2(G) + \gamma_2(\overline{G}) \leq n + 1.$$

We close this section with presenting for the Nordhaus-Gaddum-type result for the stability of 2-domination number.

Theorem 3.7. *If G is a graph of order $n \geq 2$, then*

$$st_{\gamma_2}(G) + st_{\gamma_2}(\overline{G}) \leq 2n.$$

Proof. We have $\gamma_2(G) + \gamma_2(\overline{G}) \geq 2$. Using the stability bound in Theorem 3.4 $st_{\gamma_2}(G) \leq n - \gamma_2(G) + 1$. It follows that:

$$\begin{aligned} st_{\gamma_2}(G) + st_{\gamma_2}(\overline{G}) &\leq (n - \gamma_2(G) + 1) + (n - \gamma_2(\overline{G}) + 1) \\ &= 2n + 2 - (\gamma_2(G) + \gamma_2(\overline{G})) \\ &\leq 2n. \end{aligned}$$

□

4. Results for some operations of two graphs

In this section, we study the stability of 2-domination number of some operations of two graphs. First, we consider the join of two graphs. The join $G \vee H$ of two graphs G and H with disjoint vertex sets $V(G)$ and edge sets $E(G)$ is the graph union $G \cup H$ together with all the edges joining $V(G)$.

Theorem 4.1. *If G and H are nonempty graphs, then*

$$\gamma_2(G \vee H) = \min\{\gamma_2(G), \gamma_2(H)\}.$$

Proof. Suppose that $|V(G)| = n_1$ and $|V(H)| = n_2$. It is clear to achieve that $\gamma_2(G \vee H) \geq 2$. Let $1 \leq i \leq n_1 + n_2$. We can see that for every $D_1 \subseteq V(G)$ and $D_2 \subseteq V(H)$ such that $|D_1| = i_1$ and $|D_2| = i_2$ where $i_1 + i_2 = i$, $D_1 \cup D_2$ is a 2-dominating set of $G \vee H$. Moreover, if D is a 2-dominating set for G or H of size i then D is the 2-dominating set for $G \vee H$. Therefore we have the result. □

By Theorem 4.1, we have the following result.

Theorem 4.2. *Let G and H be two nonempty graphs, then*

$$st_{\gamma_2}(G \vee H) \leq \min\{st_{\gamma_2}(G),\, st_{\gamma_2}(H)\}.$$

Here, we recall the definition of the lexicographic product of two graphs. For two graphs G and H, let $G[H]$ be the graph with vertex set $V(G) \times V(H)$ and such that vertex (a, x) is adjacent to vertex (b, y) if and only if a is adjacent to b (in G) or $a = b$ and x is adjacent to y (in H). The graph $G[H]$ is the lexicographic product (or composition) of G and H, and can be thought of as the graph arising from G and H by substituting a copy of H for every vertex of G [21].

The following theorem gives the 2-domination number of $G[H]$.

Theorem 4.3. *If G and H are two nonempty graphs. Then*

$$\gamma_2(G[H]) \leq |V(H)| \cdot \gamma_2(G).$$

Proof. Let D_G be a minimum 2-dominating set of G of size $\gamma_2(G)$. Consider the lexicographic product $G[H]$. Construct the set $D = D_G \times V(H)$, which includes all vertices in the copies of H corresponding to vertices in D_G. Since $|D| = |V(H)| \cdot \gamma_2(G)$, we only need to check that D is 2-dominating in $G[H]$.

For any vertex $(x, y) \in V(G[H])$, if $x \in D_G$ then $(x, y) \in D$. Otherwise, since D_G is 2-dominating in G, x has at least two neighbors in D_G, say g_1 and g_2. Then (x, y) is adjacent to every vertex in the copies H_{g_1} and H_{g_2}, which are subsets of D. Thus, (x, y) has at least two neighbors in D. Hence, D is a 2-dominating set of $G[H]$ and

$$\gamma_2(G[H]) \leq |D| = |V(H)| \cdot \gamma_2(G).$$

□

By Theorem 4.3, we have the following result.

Corollary 4.4. *Let G and H be two nonempty graphs. Then*

$$st_{\gamma_2}(G[H]) = \begin{cases} st_{\gamma_2}(G), & \text{if } G \text{ has no isolated vertex,} \\ \min\{st_{\gamma_2}(G), st_{\gamma_2}(H)\}, & \text{if } G \text{ has at least one isolated vertex.} \end{cases}$$

Now, we obtain the stability of 2-domination number of the corona of two graphs. We first state and prove the following theorem.

Theorem 4.5. ([16]). *Suppose that G is a graph of order n and H is any graph with no universal vertex. Then, $\gamma_2(G \circ H) = |V(G)| + \gamma_2(H)$.*

Proof. Take the vertex set $V(G)$ together with a minimum 2-dominating set of H in one copy. This forms a 2-dominating set for $G \circ H$. Obviously this set is a 2-dominating set with minimum size. So we have the result. □

Remark 1. If H contains a universal vertex (i.e., a vertex adjacent to all others in H), then $\gamma_2(G \circ H) = n$.

By Theorem 4.5, we have the following result.

Corollary 4.6. *If G and H are two graphs, then $st_{\gamma_2}(G \circ H) = 1$.*

Proof. By Theorem 4.5, $\gamma_2(G \circ H) = |V(G)| + \gamma_2(H)$, so removing any vertex from the corona product either disconnects a root vertex in G or breaks the 2-dominating set in a copy of H, thus changing $\gamma_2(G \circ H)$. Hence, we have the result. □

5. Conclusion

This paper introduces the concept of the stability of 2-domination number of a graph and explores various properties related to this number. We have determined the precise values of stability of 2-domination number for specific graphs. There is much work to be done in this area.

1. Define the edge stability of 2-domination number and study its properties.
2. What is the stability of 2-domination number of natural and fractional powers of a graph?
3. Study the complexity of the stability of 2-domination number for many of the graphs.

Conflicts of Interest. The authors declare that they have no conflicts of interest regarding the publication of this article.

Acknowledgements. The authors would like to express their gratitude to the referees for their careful reading and helpful comments.

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