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k-Fibonacci and k-Lucas Differential Equations with New Spirals

*Mehmet Pakdemirli** and *İhsan T. Dolapci*

Abstract

Fibonacci sequences and the spirals formed by employing them have found vast applications in creations and natural phenomena. In this study, new k-Fibonacci and k-Lucas differential equations are proposed. First, the k-Fibonacci and k-Lucas sequences are expressed as difference-differential equations. Then, from the difference-differential equations, the associated continuous differential equations are derived, which are linear second-order differential equations. The initial conditions for the differential equations are written with inspiration from the k-Fibonacci and k-Lucas sequences. The solutions, which are new spirals, are expressed in polar form. The spirals produce approximately the k-Fibonacci and k-Lucas numbers at constant steps of angular displacements.

Keywords: k-Fibonacci sequences, k-Lucas sequences, Continuous systems.

2020 Mathematics Subject Classification: 11B39; 34A05; 34A30; 51A50.

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1. Introduction

The k-Fibonacci and k-Lucas sequences are the generalizations of the well-known Fibonacci and Lucas sequences, which are widely used in applied mathematics. Especially, the Fibonacci sequence has been applied extensively to natural phenomena for understanding the hidden design behind it [1]. One of the characteristics

*Corresponding author (E-mail: pakdemirli@gmail.com)
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of the sequence is that the ratio of the $(n+1)$th term to the nth term approaches the limit called the golden ratio [2]. Fibonacci spirals are based mainly on this golden ratio. The classical way of constructing a Fibonacci spiral is as follows: First, a rectangle with sides in the golden ratio is taken, and a quarter circular arc is drawn starting from one of the corners in the embedded square with sides having the shorter side of the rectangular envelope. Then another quarter-circular arc, with a radius obtained by dividing the previous one by the golden ratio, is drawn. The process is repeated until the spirals converge to a point. The spirals are inscribed in the golden rectangle. Abundant applications of the spirals to galaxies, flower heads, storms, seashells, aesthetics, the human body, and architectural designs can be seen on the websites.

Fibonacci and Lucas sequences have the same formulation with different initial conditions. The properties of these sequences were extensively studied in the literature. New identities containing Fibonacci and Lucas numbers were given, and based on the new identities, the divisibility properties of Fibonacci and Lucas numbers were discussed [3]. Some new identities for the more general forms, the k-Fibonacci and k-Lucas sequences, were given by [4]. An extensive treatment of k-Fibonacci and k-Lucas sequences and their properties are due to [5].
This work is much different than the previously mentioned work in the sense that continuous solutions are produced instead of the discrete solutions of the sequences. k-Fibonacci and k-Lucas sequence formulations are the starting point in the analysis. From the formulations, k-Fibonacci and k-Lucas difference-differential equations are constructed first. The difference-differential equation form makes it possible to write the k-Fibonacci and k-Lucas differential equations, which are the main contributions of this study. Under suitable initial conditions inspired by the k-Fibonacci and k-Lucas sequences, the differential equations are solved. Using the limiting ratios of the sequences, constant angle steps are calculated. Each angle step then produces the k-Fibonacci and k-Lucas numbers similar to Binet's formula with small deviations. The differential equations presented in this work do not resemble the second-order variable coefficients differential equations that produce Fibonacci polynomials [6, 7]. Fibonacci polynomials are special polynomials that are constructed from a recurrence relation of a linear polynomial with coefficients being the Fibonacci numbers. The Fibonacci polynomial differential equation produces solutions of the form of Fibonacci polynomials, whereas the differential equations derived here lead to k-Fibonacci and k-Lucas numbers at some discrete points. The work is an extension of the previous work on the Fibonacci differential equation [8].

The solutions are expressed in polar coordinates in the form of spirals, which have a continuous change in the radius of curvature and which yield the appropriate numbers at each angular step of rotation. In contrast, the standard Fibonacci spirals have piecewise constant curvatures. The new spirals may be employed to express the natural phenomena that need extensive analysis and are left as a further topic of research.

2. Preliminary information on k-Fibonacci and k-Lucas sequences

In this section, the preliminary information on k-Fibonacci and k-Lucas sequences will be given.

2.1 k-Fibonacci sequences

k-Fibonacci numbers are defined as discrete equations

$$F_{n+2}^{(k)}= kF_{n+1}^{(k)}+F_{n}^{(k)}, \quad n = 1, 2, 3\dots \tag{1}$$

with initial values

$$F_1^{(k)}= 1, \quad F_2^{(k)}= k, \tag{2}$$

with $k = 1$ being the well-known Fibonacci sequence and $k = 2$ corresponding to the Pell-Fibonacci numbers. The numbers corresponding to the first three k values are

$$F_n^{(1)}= \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots.\}, \tag{3}$$

$$F_n^{(2)}= \{1, 2, 5, 12, 29, 70, 169, 408, \dots.\}, \tag{4}$$

$$F_n^{(3)}= \{1, 3, 10, 33, 109, 360, 1189, 3927, \dots.\}. \tag{5}$$

The general formulas for the first 8 numbers in terms of the parameter k are [5]:

$$F_1^{(k)}= 1, F_2^{(k)}= k, F_3^{(k)}=k^2+1, F_4^{(k)}=k^3+2k, F_5^{(k)}=k^4+3k^2+1,$$

$$F_6^{(k)}=k^5+4k^3+3k, F_7^{(k)}=k^6+5k^4+6k^2+1, F_8^{(k)}=k^7+6k^5+10k^3+4k. \tag{6}$$

Assuming a solution for Equation (1), $F_n^{(k)}=r^n$, which reduces to $r^2-kr-1 = 0$ upon simplification, with a solution of the roots

$$r_1=\frac{k+\sqrt{k^2+4}}{2}, \quad r_2=\frac{k-\sqrt{k^2+4}}{2}. \tag{7}$$

Therefore,

$$r =c_1\left(\frac{k-\sqrt{k^2+4}}{2}\right)^n+c_2\left(\frac{k+\sqrt{k^2+4}}{2}\right)^n. \tag{8}$$

For the initial conditions, n=0, r=0 and for n=1, r=1, the coefficients turn out to be, $c_1= -\frac{1}{\sqrt{k^2+4}}$, $c_2=\frac{1}{\sqrt{k^2+4}}$, hence the k-Fibonacci numbers are:

$$F_n^{(k)}= -\frac{1}{\sqrt{k^2+4}}\left(\frac{k-\sqrt{k^2+4}}{2}\right)^n+\frac{1}{\sqrt{k^2+4}}\left(\frac{k+\sqrt{k^2+4}}{2}\right)^n. \tag{9}$$

Defining

$$\varphi_k = \frac{k+\sqrt{k^2+4}}{2}, \ \lambda_k = \frac{k-\sqrt{k^2+4}}{2}, \tag{10}$$

and noting that $\varphi_k \lambda_k = -1$, the k-Fibonacci numbers are expressed in a more compact form

$$F_n^{(k)} = \frac{1}{\sqrt{k^2+4}} \left[(\varphi_k)^n - \left(-\frac{1}{\varphi_k} \right)^n \right]. \tag{11}$$

The positive roots have special names

$$\varphi_1 = \frac{1+\sqrt{5}}{2}, \quad \text{(Golden Ratio)}, \tag{12}$$

$$\varphi_2 = 1+\sqrt{2}, \quad \text{(Silver Ratio)}, \tag{13}$$

$$\varphi_3 = \frac{3+\sqrt{13}}{2}, \quad \text{(Bronze Ratio)}. \tag{14}$$

The ratio of the $(n+1)$th term to the nth term as n approaches infinity is

$$\lim_{n\to\infty} \frac{F_{n+1}^{(k)}}{F_n^{(k)}} = \varphi_k. \tag{15}$$

2.2 k-Lucas sequences

k-Lucas numbers are defined similar to the k-Fibonacci numbers

$$L_{n+2}^{(k)} = kL_{n+1}^{(k)} + L_n^{(k)}, \quad n = 0, 1, 2, 3 \ldots \tag{16}$$

with the initial values different from the Fibonacci numbers

$$L_0^{(k)} = 2, \quad L_1^{(k)} = k. \tag{17}$$

Note that $k = 1$ corresponds to the well-known Lucas numbers and k = 2 to the Pell-Lucas numbers:

$$L_n^{(1)} = \{2, 1, 3, 4, 7, 11, 18, 29, 47, 76 \ldots .\}, \tag{18}$$

$$L_n^{(2)} = \{2, 2, 6, 14, 34, 82, 198, 478 \ldots .\}, \tag{19}$$

$$L_n^{(3)} = \{2, 3, 11, 36, 119, 393, 1298, 4287 \ldots .\}. \tag{20}$$

The first eight k-Lucas numbers, where k is an arbitrary value, are [5]:

$$L_0^{(k)} = 2, L_1^{(k)} = k, L_2^{(k)} = k^2+2, L_3^{(k)} = k^3+3k, L_4^{(k)} = k^4+4k^2+2,$$

$$L_5^{(k)} = k^5+5k^3+5k, L_6^{(k)} = k^6+6k^4+9k^2+2, L_7^{(k)} = k^7+7k^5+14k^3+7k. \tag{21}$$

Assuming a similar solution as in Section 2.1 and imposing the conditions $n = 0, r = 2$ and $n = 1, r = k$, the solution turns out to be

$$L_n^{(k)}=\left(\frac{k-\sqrt{k^2+4}}{2}\right)^n+\left(\frac{k+\sqrt{k^2+4}}{2}\right)^n, \tag{22}$$

or using Equation (10)

$$L_n^{(k)}=\left(-\frac{1}{\varphi_k}\right)^n+(\varphi_k)^n. \tag{23}$$

Similar to the k-Fibonacci sequences, the ratio of the $(n+1)$th term to the nth term as n approaches infinity is

$$\lim_{n\to\infty}\frac{L_{n+1}^{(k)}}{L_n^{(k)}}=\varphi_k, \tag{24}$$

which is independent of the initial conditions.

3. Derivation of the differential equations

The discrete k-Fibonacci and k-Lucas difference equations will be transformed into continuous differential equations. This is done by writing the associated difference-differential equation as an intermediate step and then using the analogy between the difference-differential equations and continuous differential equations to write the final differential equation.

Difference equations are discrete without producing continuous solutions. They generate numbers at some isolated points. In contrast, the differential equations lead to solutions that are continuous. The discrete-differential and continuous differential equations are connected to each other. Hence, for every discrete-differential equation, a corresponding continuous differential equation can be written and vice versa [9]. The well-known numerical methods such as the Euler method, the Runge-Kutta method, and the Finite Element method produce discrete solutions at some steps of integration of the continuous differential equations. Using a small step size produces solutions that appear continuous graphically, even though they are discrete. Another example of difference equations is the recurrence relations appearing in series solutions of differential equations (Frobenius method). Generally speaking, continuous solutions are available if the differential equation possesses an exact analytical solution or an approximate analytical solution. Our goal will be to write the difference-differential equation associated with the discrete sequence equation of the k-Fibonacci and k-Lucas numbers and switch to the continuous form using the analogy.

3.1 k-Fibonacci differential equation

The difference operators are [9]:

$$D(r_n) = r_{n+1} - r_n, \quad D^2(r_n) = r_{n+2} - 2r_{n+1} + r_n. \tag{25}$$

To express (1) in terms of difference operators

$$D^2(r_n) + \alpha D(r_n) + \beta r_n = r_{n+2} - kr_{n+1} - r_n, \tag{26}$$

for some coefficients α and β. Substituting (25) into (26) and equating like terms at both sides

$$\alpha = 2 - k, \beta = -k. \tag{27}$$

The difference equation is then

$$D^2(r_n) + (2-k)D(r_n) - kr_n = 0. \tag{28}$$

The associated continuous differential equation can be immediately written from analogy as:

$$\frac{d^2r}{d\theta^2} + (2-k)\frac{dr}{d\theta} - kr = 0. \tag{29}$$

In writing the analogy, first-order Taylor expansions of the continuous derivatives are considered in the discrete derivatives following [8]. The coefficients of the differential equation are also first-order approximations in the transformed case. The equation is a constant coefficient linear equation whose solution is:

$$r(\theta) = c_1 \exp\left(\frac{k-2-\sqrt{k^2+4}}{2}\theta\right) + c_2 \exp\left(\frac{k-2+\sqrt{k^2+4}}{2}\theta\right). \tag{30}$$

With analogy from the initial conditions of the original sequence $F_1^{(k)} = 1,\ F_2^{(k)} = k$, the conditions for the differential equation can be written as:

$$r(0) = 1, \frac{dr}{d\theta}(0) = k - 1, \tag{31}$$

which is again a first-order approximation of the derivatives. The conditions yield $c_1 = \frac{\sqrt{k^2+4}-k}{2\sqrt{k^2+4}}$, $c_2 = \frac{\sqrt{k^2+4}+k}{2\sqrt{k^2+4}}$. Hence the solution is:

$$r(\theta) = \frac{\sqrt{k^2+4}-k}{2\sqrt{k^2+4}} \exp\left(\frac{k-2-\sqrt{k^2+4}}{2}\theta\right) + \frac{\sqrt{k^2+4}+k}{2\sqrt{k^2+4}} \exp\left(\frac{k-2+\sqrt{k^2+4}}{2}\theta\right), \tag{32}$$

or

$$r(\theta) = \frac{1}{\varphi_k\sqrt{k^2+4}} \exp\left(-\left(\frac{1}{\varphi_k}+1\right)\theta\right) + \frac{\varphi_k}{\sqrt{k^2+4}} \exp\left((\varphi_k - 1)\theta\right). \tag{33}$$

The solution expressed in polar form is continuously changing. In order to have the k-Fibonacci numbers at each constant step of angle, choose $\theta = (n-1)\theta_0$ for a constant value of θ_0. One may look for the specific θ_0 for which the below limit equals

$$\lim_{n\to\infty} \frac{r((n+1)\,\theta)}{r(n\theta)} = \varphi_k \ . \tag{34}$$

The result is

$$\theta_{0k} = \frac{ln\varphi_k}{\varphi_k - 1}. \tag{35}$$

3.2 k-Lucas differential equation

Derivation of the differential equation associated with k-Lucas numbers is essentially the same as the previous section. The differential equation is the same equation

$$\frac{d^2r}{d\theta^2} + (2-k)\frac{dr}{d\theta} - kr = 0, \tag{36}$$

having different initial conditions written with inspiration from the conditions $L_0^{(k)} = 2,\ \ L_1^{(k)} = k$

$$r\,(0) = 2, \quad \frac{dr}{d\theta}\,(0) = k-2. \tag{37}$$

The solution is

$$r\,(\theta) = \exp\left(\frac{k-2-\sqrt{k^2+4}}{2}\theta\right) + \exp\left(\frac{k-2+\sqrt{k^2+4}}{2}\theta\right), \tag{38}$$

or

$$r\,(\theta) = \exp\left(-\left(\frac{1}{\varphi_k}+1\right)\theta\right) + \exp\left((\varphi_k - 1)\,\theta\right). \tag{39}$$

Choose $\theta = n\theta_0$ for a constant value angular step size θ_0 to determine the Lucas numbers at each multiple of the step size. θ_{0k} is the same equation given in (35).

4. Spiral solutions

The solutions expressed in polar coordinates are spirals. The distance from the origin expressed by r gives the sequence numbers approximately at each calculated angular step. Numerical results for $k = 1, 2, 3$ are given, although the analysis can be repeated for any arbitrary k values.

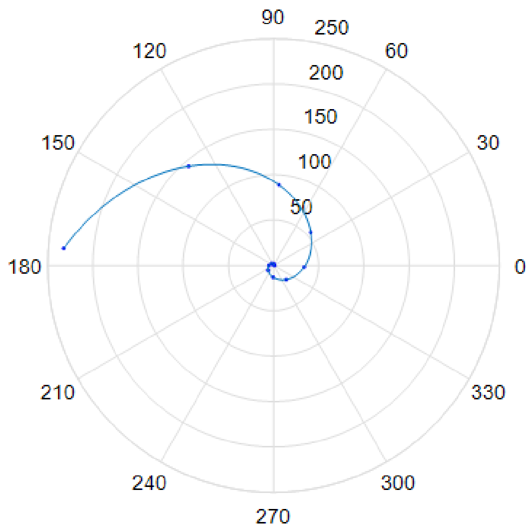

Figure 1: $k = 1$ Fibonacci spiral (13 terms).

4.1 k-Fibonacci spirals

The spirals are depicted in Figures 1 to 3 for $k = 1, 2, 3$, respectively. Dots represent the numerical values calculated from the original sequences. They are placed at each constant angular step size calculated from (35)

$$\theta_{01} = 0.778617, \theta_{02} = 0.623225, \theta_{03} = 0.518836, \tag{40}$$

with the distances to the origin representing the numerical values of the sequence.

Since the numbers grow rapidly, for higher k values, fewer terms are taken in the figures so that the initial numbers can also be visualized. The normalized spirals can be scaled with an arbitrary r_0 scaling factor for appropriate applications.

The spirals shown here can be represented by a simple function of the form $r = r(\theta)$ with referral to a single origin. The classical Fibonacci spirals have a constant radius of curvature for every 90 degrees of rotation and cannot be expressed with reference to a single origin.

The exact values calculated from the original sequences are contrasted with the approximate values calculated from the spirals (Tables 1 to 3). As a general rule, with some exceptions, the percentage error decreases, and the match becomes better as n increases. The high discrepancy for $n = 2$ stems from the fact that the derivatives are replaced by first-order truncations, and a linear approximation of the initial slope is the initial condition. As the number k increases, from Tables 1 to 3, it is evident that the errors substantially decrease.

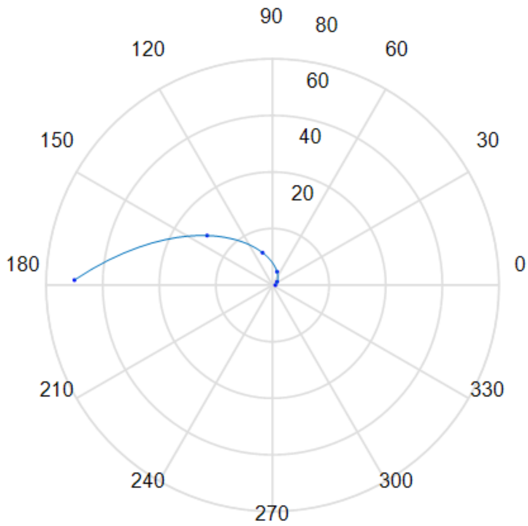

Figure 2: $k = 2$ Fibonacci spiral (6 terms).

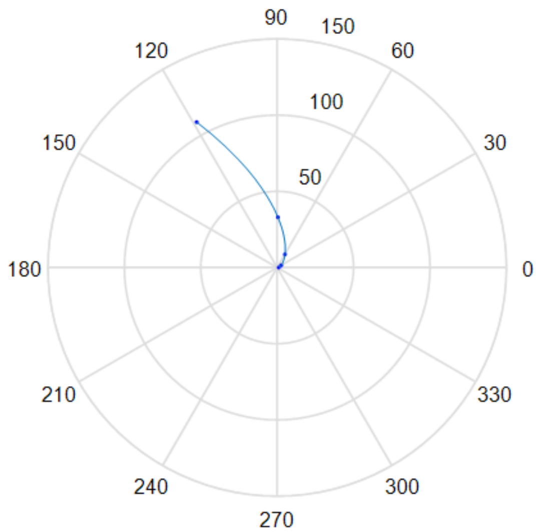

Figure 3: $k = 3$ Fibonacci spiral (5 terms).

Table 1: Comparison of the Fibonacci numbers and spiral solutions ($k = 1$).

n	$r_n = r((n-1)\,\theta_0)$ ($\theta_0 = 0.778617$)	$F_n^{(1)}$	$\%e_n = \left\lvert \frac{r_n - F_n^{(1)}}{F_n^{(1)}} \right\rvert x100$
1	1.00000	1	0
2	1.24923	1	24.92
3	1.91667	2	4.17
4	3.07156	3	2.39
5	4.96146	5	0.77
6	8.02543	8	0.32
7	12.9847	13	0.12
8	21.0096	21	0.05
9	33.9941	34	0.02
10	55.0036	55	0.01
11	88.9977	89	0.002
12	144.001	144	0.0006
13	232.999	233	0.0004
14	377.000	377	0

Despite the disadvantages of predicting Fibonacci numbers with relatively high errors for $n = 2$, the advantages of the continuous approach may be as follows: *i*) The solution functions possess differentiability/integrability properties. *ii*) The curvatures are continuously changing for spirals which display a better visualization and a possibly better approximation for real problems. *iii*) The continuous approach makes it possible to interpolate between the integer values.

4.2 k-Lucas spirals

The Lucas spirals are given in Figures 4 to 6 for $k = 1, 2, 3$, respectively. The original values calculated from the sequences are indicated as dots in the figures. The constant angular step sizes for which the spiral gives the sequence numbers are the same as in the previous section, see Equation (40). To visualize the initial values properly, fewer terms are taken in the spirals as k increases. The exact values calculated from the original Lucas sequences are contrasted with the approximate values calculated from the k-Lucas spirals (Tables 4 to 6). Similar to the Fibonacci case, the percentage error decreases, and the match becomes better as n increases, with some exceptions. The high discrepancy for $n = 1$ and 2 stems from the fact that the derivatives are replaced by first-order approximations and the initial slope is given as a first-order linear approximation for the differential equation. As k increases, the error for each number decreases also.

Table 2: Comparison of the Fibonacci numbers and spiral solutions ($k = 2$).

n	$r_n = r((n-1)\,\theta_0)$ ($\theta_0 = 0.623225$)	$F_n^{(2)}$	$\%e_n = \left\lvert \frac{r_n - F_n^{(2)}}{F_n^{(2)}} \right\rvert x100$
1	1.00000	1	0
2	2.12132	2	6.066
3	5.00000	5	0
4	12.0208	12	0.173
5	29.0000	29	0
6	70.0035	70	0.005
7	169.000	169	0
8	408.000	408	0
9	984.997	985	0.0003
10	2377.99	2378	0.0004
11	5740.98	5741	0.0003
12	13859.9	13860	0.0007
13	33460.9	33461	0.0002
14	80781.6	80782	0.0004

Table 3: Comparison of the Fibonacci numbers and spiral solutions ($k = 3$).

n	$r_n = r((n-1)\,\theta_0)$ ($\theta_0 = 0.518836$)	$F_n^{(3)}$	$\%e_n = \left\lvert \frac{r_n - F_n^{(3)}}{F_n^{(3)}} \right\rvert x100$
1	1.00000	1	0
2	3.06814	3	2.271
3	10.0140	10	0.14
4	33.0134	33	0.04
5	109.005	109	0.004
6	360.003	360	0.0008
7	1189.00	1189	0
8	3926.99	3927	0.0002
9	12970.0	12970	0
10	42836.9	42837	0.0002
11	141481	141481	0

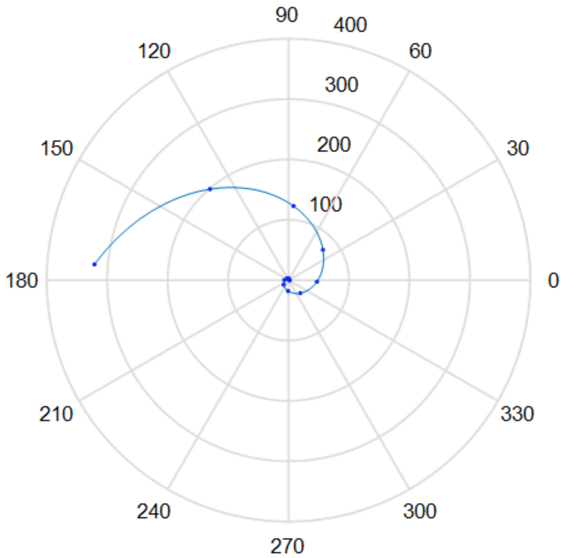

Figure 4: $k = 1$ Lucas spiral (13 terms).

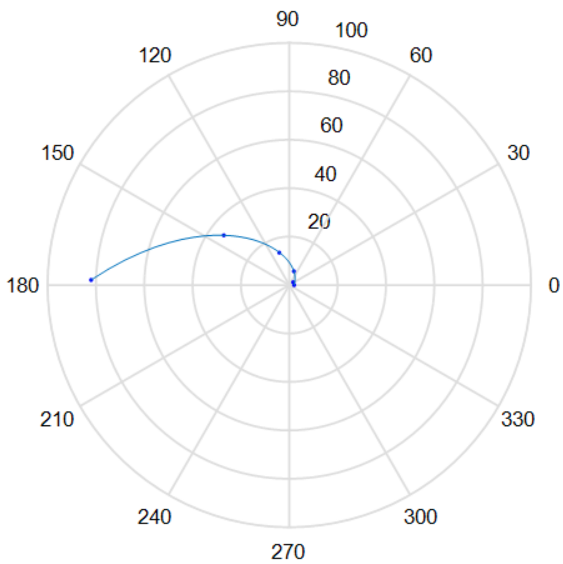

Figure 5: $k = 2$ Lucas spiral (6 terms).

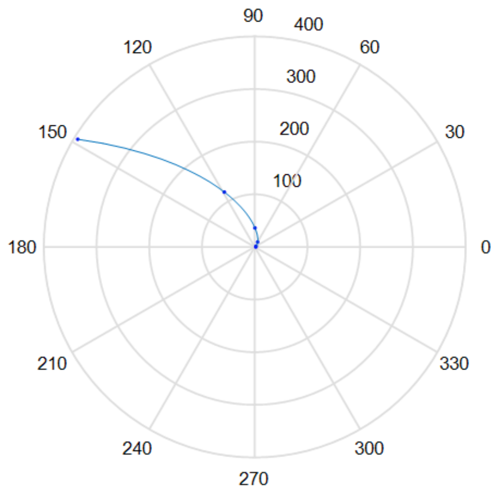

Figure 6: $k = 3$ Lucas spiral (6 terms).

Table 4: Comparison of the Lucas numbers and spiral solutions ($k = 1$).

n	$r_n = r(n\theta_0)$ ($\theta_0 = 0.778617$)	$L_n^{(1)}$	$\%e_n = \left\lvert \frac{r_n - L_n^{(1)}}{L_n^{(1)}} \right\rvert x100$
0	2.00000	2	0
1	1.90174	1	90.174
2	2.69852	3	10.0493
3	4.25890	4	6.4725
4	6.86058	7	1.99
5	11.0920	11	0.836
6	17.9448	18	0.3067
7	29.0346	29	0.12
8	46.9787	47	0.04
9	76.0131	76	0.017
10	122.992	123	0.006
11	199.005	199	0.002
12	321.997	322	0.0009
13	521.002	521	0.0002

Table 5: Comparison of the Lucas numbers and spiral solutions ($k = 2$).

n	$r_n = r(n\theta_0)$ ($\theta_0 = 0.623225$)	$L_n^{(2)}$	$\%e_n = \left\lvert \frac{r_n - L_n^{(2)}}{L_n^{(2)}} \right\rvert x100$
0	2.00000	2	0
1	2.82843	2	41.42
2	6.00000	6	0
3	14.1421	14	1.015
4	34.0000	34	0
5	82.0242	82	0.0295
6	198.000	198	0
7	478.003	478	0.0006
8	1154.00	1154	0
9	2785.99	2786	0.0003
10	6725.98	6726	0.0003
11	16237.9	16238	0.0003
12	39201.8	39202	0.0004
13	94641.6	94642	0.0004

Table 6: Comparison of the Lucas numbers and spiral solutions ($k = 3$).

n	$r_n = r((n-1)\,\theta_0)$ ($\theta_0 = 0.518836$)	$F_n^{(3)}$	$\%e_n = \left\lvert \frac{r_n - F_n^{(3)}}{F_n^{(3)}} \right\rvert x100$
0	2.00000	2	0
1	3.81146	3	27.0486
2	11.1671	11	1.519
3	36.1593	36	0.44
4	119.058	119	0.04
5	393.036	393	0.009
6	1298.01	1298	0.001
7	4287.00	4287	0
8	14159.0	14159	0
9	46763.9	46764	0.0002
10	154451.0	154451	0.0003

5. Concluding remarks

Starting from the discrete equations of k-Fibonacci and k-Lucas sequences, the continuous equations in the form of differential equations are proposed for the first time. The sequences are written in the form of discrete-differential equations and then by analogy, the differential equations are written. The equations are solved with initial conditions inspired from the original sequences. The solutions of the differential equations in polar form lead to new spirals which are different than the classical Fibonacci spirals. At constant steps of rotation, the spirals yield the k-Fibonacci and k-Lucas numbers approximately. The discrepancies between the numbers for the leading numbers in the sequences stem from the first-order approximations in the derivatives and linear approximations in the initial slopes used in the differential equation systems. Such errors vanish as the number of terms in the sequence increases. Despite the above mentioned disadvantage, the new continuous approach has several advantages, such as analytical properties (differentiability, integration), better visualization and adaptation to continuously changing curvatures and interpolation between the integer values.

The new spirals presented in this work may find applications in explaining the geometrical patterns observed in nature. This task requires, however further experimental and analytical studies, which are left for future research.

Conflicts of Interest. The authors declare that they have no conflicts of interest regarding the publication of this article.

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Mehmet Pakdemirli
Mechanical Engineering Department,
Manisa Celal Bayar University,
Muradiye, Yunusemre, Manisa, Turkey
e-mail: pakdemirli@gmail.com

İhsan T. Dolapci
Mechanical Engineering Department,
Manisa Celal Bayar University,
Muradiye, Yunusemre, Manisa, Turkey
e-mail: ihsandolapci@gmail.com