Mathematics Interdisciplinary Research **3** (2018) 55 – 65

Some Applications of Strong Product

Mostafa Tavakoli ⋆, *Freydoon Rahbarnia and*
Irandokht Rezaee Abdolhosseinzadeh

Abstract

Let G and H be two graphs. The strong product $G \boxtimes H$ of the graphs G and H is the graph with vertex set $V(G) \times V(H)$, and $u = (u_1, v_1)$ is adjacent with $v = (u_2, v_2)$ whenever ($v_1 = v_2$ and u_1 is adjacent with u_2) or ($u_1 = u_2$ and v_1 is adjacent with v_2) or (u_1 is adjacent with u_2 and v_1 is adjacent with v_2). In this paper, some applications of this product are presented. Finally, we pose one open problem related to this topic.

Keywords: Strong product, graph invariant, fence graph.

2010 Mathematics Subject Classification: Primary: 05C76, Secondary: 05C12, 05C07.

How to cite this article
M. Tavakoli, F. Rahbarnia and I. Rezaee Abdolhosseinzadeh, Some applications of strong product, *Math. Interdisc. Res.* **3** (2018) 55-65.

1. Introduction

A **topological index** is a real number related to a graph. It must be a structural invariant, i.e., it preserves by every graph automorphism. A topological index is a graph invariant applicable in chemistry. Suppose G is a graph with the vertex and edge sets of $V(G)$ and $E(G)$, respectively. If $x, y \in V(G)$, then the **distance** $d_G(x, y)$ (or $d(x, y)$ for short) between x and y is defined as the length of a minimum path connecting x and y. The **Wiener index** of G, $W(G)$, is defined as the summation of distances between all pairs of vertices in G. In other words, the Wiener index of a graph G is defined as $W(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u, v)$ [21]. A topological index is called distance-based if it can be defined by the distance function $d(-, -)$. It is worthy to mention here that Wiener did not consider the

⋆Corresponding author (Email: m_tavakoli@um.ac.ir)
Academic Editor: Gholam Hossein Fath-Tabar
Received 01 June 2016, Accepted 03 April 2018
DOI: 10.22052/mir.2018.55115.1033
©2018 University of Kashan
This work is licensed under the Creative Commons Attribution 4.0 International License.

distance function $d(-,-)$ in the seminal paper. Hosoya [12], presented a new simple formula for the Wiener index by using distance function. We encourage the readers to consult [6,7] for more information on Wiener index.

The **hyper-Wiener index** of acyclic graphs was introduced by Milan Randić in 1993. Then Klein et al. [16], generalized Randić's definition for all connected graphs, as a generalization of the Wiener index. It is defined as

$$WW(G) = \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} (d(u,v) + d^2(u,v))$$

or

$$WW(G) = \frac{1}{2}W(G) + \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} d^2(u,v).$$

The mathematical properties and chemical meaning of this topological index are reported in [4,5,9,15,25].

As usual, the **degree** of a vertex u of G is denoted by $deg(u)$ and it is defined as the number of edges incident with u. The **Zagreb indices** have been introduced more than thirty years ago by Gutman and Trinajstić, [10]. They are defined as:

$$M_1(G) = \sum_{u \in V(G)} deg(u)^2,$$
$$M_2(G) = \sum_{uv \in E(G)} deg(u)deg(v).$$

We encourage the reader to consult [1,10,23] for historical background, computational techniques and mathematical properties of Zagreb indices.

The **eccentricity** $\varepsilon_G(u)$ is defined as the largest distance between u and other vertices of G. We will omit the subscript G when the graph is clear from the context. The **eccentric connectivity** index of a graph G is defined as $\xi^c(G) = \sum_{u \in V(G)} deg_G(u)\varepsilon_G(u)$ [19]. We encourage the reader to consult the papers [2,3] for some applications and the papers [13,17,22,24] for the mathematical properties of this topological index. For a given vertex $u \in V(G)$ we define its **distance sum** $D_G(u)$ as $D_G(u) = \sum_{v \in V(G)} d_G(u,v)$. The **eccentric distance sum** of G is summation of all quantity $D_G(u)\varepsilon_G(u)$ over all vertices of G [8]. In other words, $\xi^{SD}(G) = \sum_{u \in V(G)} D_G(u)\varepsilon_G(u)$. The concept of eccentricity also gives rise to a number of other topological invariants. For example, the **total eccentricity** $\zeta(G)$ of a graph G is defined as $\zeta(G) = \sum_{u \in V(G)} \varepsilon_G(u)$.

The ***n*-cube** $Q_n (n \geq 1)$ is the graph whose vertex set is the set of all n-tuples of 0s and 1s, where two n-tuples are adjacent if they differ in precisely one coordinate. Q_n has 2^n vertices, $2^{n-1}n$ edges, and is a regular graph with n edges touching each vertex. A graph G is called **nontrivial** if $|V(G)| > 1$. Also, we denote the path graph, the complete and the cycle of order n by P_n, K_n and C_n, respectively.

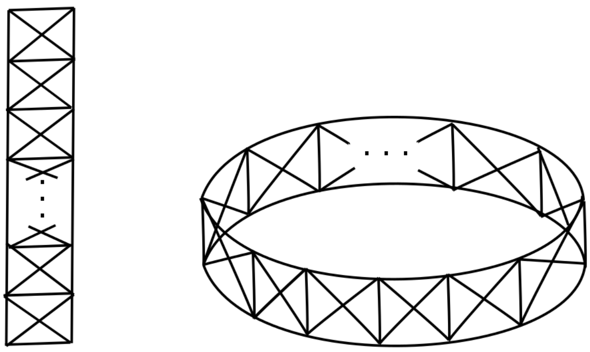

Figure 1: Open and closed fences.

The **Strong** product $G \boxtimes H$ of the graphs G and H has the vertex set $V(G \boxtimes H) = V(G) \times V(H)$ and $(a,x)(b,y)$ is an edge of $G \boxtimes H$ if $a = b$ and $xy \in E(H)$, or $ab \in E(G)$ and $x = y$, or $ab \in E(G)$ and $xy \in E(H)$. Occasionally one also encounters the names strong direct product or symmetric composition for the strong product [11]. As an example, see open and closed fences, $P_n \boxtimes K_2$, $C_n \boxtimes K_2$, Figure 1.

2. Results

For a connected graph G, the radius $r(G)$ and diameter $D(G)$ are, respectively, the minimum and maximum eccentricity among the vertices of G.

Lemma 2.1. [20] *Let G and H be two graphs. Then for every vertex (a,x) of $G \boxtimes H$, we have*

$$\varepsilon_{G\boxtimes H}((a,x)) = max\{\varepsilon_G(a), \varepsilon_H(x)\}.$$

Theorem 2.2. [20] *Let G and H be nontrivial connected graphs. Then $G \boxtimes H$ is eulerian if and only if G and H are eulerian.*

By the above theorem, $C_n \boxtimes C_m$ and $K_{2n+1} \boxtimes C_m$ are eulerian.

Theorem 2.3. [20] *Let G and H be nontrivial connected graphs. Then*

$$\begin{aligned} W(G \boxtimes H) \geq\ & (|V(G)| + 2|E(G)|)W(H) + (|V(H)| + 2|E(H)|)W(G) \\ & + |V(G)||V(H)|(|V(G)||V(H)| - |V(G)| - |V(H)| + 1) \\ & - 2|E(G)||V(H)|(|V(H)| - 1) - 2|E(H)|(|V(G)|^2|V(G)| - |E(G)|), \end{aligned}$$

with equality if and only if $max\{D(G), D(H)\} \leq 2$, or $G \cong K_n$, or $H \cong K_n$.

We apply Theorem 2.3 to compute the Wiener index of $K_n \boxtimes C_m$ and $K_n \boxtimes P_m$. We have

$$\begin{aligned} W(K_n \boxtimes C_m) &= (|V(K_n)| + 2|E(K_n)|)W(C_m) + (|V(C_m)| + 2|E(C_m)|)W(K_n) \\ &+ |V(K_n)||V(C_m)|(|V(K_n)||V(C_m)| - |V(K_n)| - |V(C_m)| + 1) \\ &- 2|E(K_n)||V(C_m)|(|V(C_m)| - 1) \\ &- 2|E(C_m)|(|V(K_n)|^2 - |V(K_n)| - |E(K_n)|), \end{aligned}$$

on the other hand, by [18], $W(C_n) = \begin{cases} \frac{n^3}{8} & 2|n \\ \frac{n(n^2-1)}{8} & 2 \nmid n \end{cases}$ and $W(K_n) = \frac{n(n-1)}{2}$.

Using a tedious calculation, we have:

$$W(K_n \boxtimes C_m) = \begin{cases} \frac{1}{8}n^2m^3 + \frac{1}{2}n^2m - \frac{1}{2}nm & 2|m, \\ \\ \frac{1}{8}n^2m^3 + \frac{3}{8}n^2m - \frac{1}{2}nm & 2 \nmid m. \end{cases} \tag{1}$$

Also, by [18], $W(P_n) = \frac{n(n^2-1)}{6}$, then

$$\begin{aligned} W(K_n \boxtimes P_m) &= (|V(K_n)| + 2|E(K_n)|)W(P_m) + (|V(P_m)| + 2|E(P_m)|)W(K_n) \\ &+ |V(K_n)||V(P_m)|(|V(K_n)||V(P_m)| - |V(K_n)| - |V(P_m)| + 1) \\ &- 2|E(K_n)||V(P_m)|(|V(P_m)| - 1) \\ &- 2|E(P_m)|(|V(K_n)|^2 - |V(K_n)| - |E(K_n)|) \\ &= \frac{1}{6}n^2m^3 + \frac{1}{3}n^2m - \frac{1}{2}nm. \end{aligned} \tag{2}$$

By replacing n with 2 in the relations (1) and (2), we obtain W of open and closed fences, as follow:

$$W(K_2 \boxtimes C_m) = \begin{cases} \frac{1}{2}m^3 + 2m - m & 2|m, \\ \\ \frac{1}{2}m^3 + \frac{3}{2}m - m & 2 \nmid m, \end{cases}$$

$$W(K_2 \boxtimes P_m) = \frac{2}{3}m^3 + \frac{4}{3}m - m.$$

Theorem 2.4. [20] *Let G and H be nontrivial connected graphs. Then*

$$\begin{aligned} W(G\boxtimes H) &\leqslant (|V(G)|+2|E(G)|)W(H)+(|V(H)|+2|E(H)|)W(G)\\ &+ D\Big[\frac{|V(G)||V(H)|}{2}(|V(G)||V(H)|-|V(G)|-|V(H)|+1)\\ &- 2|E(H)|\binom{|V(G)|}{2}-2|E(G)|\binom{|V(H)|}{2}\Big]\\ &+ 2|E(G)||E(H)|(D-1),\end{aligned}$$

where $D = max\{D(G), D(H)\}$. Moreover, the upper bound is attained if and only if $D \leqslant 2$, or $G \cong K_n$, or $H \cong K_n$.

Theorem 2.5. [20] *Let G and H be nontrivial connected graphs. Then*

$$\begin{aligned} WW(G\boxtimes H) &\geqslant (|V(G)|+2|E(G)|)WW(H)+(|V(H)|+2|E(H)|)WW(G)\\ &+ \frac{3}{2}|V(G)||V(H)|(|V(G)||V(H)|-|V(G)|-|V(H)|+1)\\ &- 3|E(G)||V(H)|(|V(H)|-1)\\ &- 3|E(H)|(|V(G)|^2-|V(G)|-\frac{4}{3}|E(G)|),\end{aligned}$$

with equality if and only if $max\{D(G), D(H)\} \leqslant 2$, or $G \cong K_n$, or $H \cong K_n$.

We apply Theorem 2.5 to compute the hyper-Wiener index of $K_n \boxtimes C_m$ and $K_n \boxtimes P_m$. We have:

$$\begin{aligned} WW(K_n\boxtimes C_m) &= (|V(K_n)|+2|E(K_n)|)WW(C_m)\\ &+ (|V(C_m)|+2|E(C_m)|)WW(K_n)\\ &+ \frac{3}{2}|V(K_n)||V(C_m)|(|V(K_n)||V(C_m)|-|V(K_n)|-|V(C_m)|+1)\\ &- 3|E(K_n)||V(C_m)|(|V(C_m)|-1)\\ &- 3|E(C_m)|(|V(K_n)|^2-|V(K_n)|-\frac{4}{3}|E(K_n)|).\end{aligned}$$

On the other hand, by [14],

$$WW(C_n)=\begin{cases}\dfrac{n^2(n+1)(n+2)}{48} & 2|n,\\[2ex] \dfrac{n(n^2-1)(n+3)}{48} & 2\nmid n.\end{cases}$$

Using a tedious calculation, we have:

$$WW(K_n\boxtimes C_m)=\begin{cases}\dfrac{1}{48}n^2m^2(m^2+3m+2)+\dfrac{1}{2}mn(n-1) & 2|m,\\[2ex] \dfrac{1}{48}n^2m^2(m^2+3m-1)+\dfrac{1}{2}mn(\dfrac{7}{8}n-1) & 2\nmid m.\end{cases} \tag{3}$$

Also, by [14], $WW(P_n) = \frac{1}{24}(n^4 + 2n^3 - n^2 - 2n)$, then

$$\begin{aligned} WW(K_n \boxtimes P_m) &= (|V(K_n)| + 2|E(K_n)|)WW(P_m) \\ &+ (|V(P_m)| + 2|E(P_m)|)WW(K_n) \\ &+ \frac{3}{2}|V(K_n)||V(P_m)|(|V(K_n)||V(P_m)| - |V(K_n)| - |V(P_m)| + 1) \\ &- 3|E(K_n)||V(P_m)|(|V(P_m)| - 1) \\ &- 3|E(P_m)|(|V(K_n)|^2 - |V(K_n)| - \frac{4}{3}|E(K_n)|) \\ &= \frac{1}{24}m^2n^2(m^2 + 2m - 1) + \frac{1}{2}mn(\frac{5}{6}n - 1). \end{aligned} \tag{4}$$

If $n = 2$ in the relations (3) and (4), we have the hyper-Wiener index of open and closed fences, as follow:

$$WW(K_2 \boxtimes C_m) = \begin{cases} \frac{1}{12}m^2(m^2 + 3m + 2) + m & 2|m, \\ \frac{1}{12}m^2(m^2 + 3m - 1) + \frac{3}{4}m & 2 \nmid m, \end{cases}$$
$$WW(K_2 \boxtimes P_m) = \frac{1}{6}m^2(m^2 + 2m - 1) + \frac{2}{3}m.$$

Theorem 2.6. [20] *Let G and H be nontrivial connected graphs. Then*

$$\begin{aligned} WW(G \boxtimes H) &\leqslant (|V(G)| + 2|E(G)|)WW(H) + (|V(H)| + 2|E(H)|)WW(G) \\ &+ \frac{1}{2}D(D+1)\Big[\frac{|V(G)||V(H)|}{2}(|V(G)||V(H)| \\ &- |V(G)| - |V(H)| + 1) \\ &- 2|E(H)|\binom{|V(G)|}{2} - 2|E(G)|\binom{|V(H)|}{2}\Big] \\ &+ |E(G)||E(H)|(D^2 + D - 2), \end{aligned}$$

where $D = max\{D(G), D(H)\}$. Moreover, the upper bound is attained if and only if $D \leqslant 2$, or $G \cong K_n$, or $H \cong K_n$.

Theorem 2.7. [20] *For graphs G and H, we have*

$$\begin{aligned} M_1(G \boxtimes H) &= (|V(H)| + 4|E(H)|)M_1(G) + (|V(G)| + 4|E(G)|)M_1(H) \\ &+ M_1(G)M_1(H) + 8|E(G)||E(H)|. \end{aligned}$$

By the previous theorem, we have

$$
\begin{aligned}
M_1(P_n \boxtimes C_m) &= (|V(C_m)| + 4|E(C_m)|)M_1(P_n) + (|V(P_n)| + 4|E(P_n)|)M_1(C_m) \\
&\quad + M_1(P_n)M_1(C_m) + 8|E(P_n)||E(C_m)| = 64mn - 78m, \qquad (5) \\
M_1(P_n \boxtimes P_m) &= 64mn - 78n - 78m + 92, \qquad (6) \\
M_1(C_n \boxtimes C_m) &= 64nm.
\end{aligned}
$$

If $n = 2$ in the relations (5) and (6), we have M_1 of open and closed fences, as follow:

$$
\begin{aligned}
M_1(P_2 \boxtimes C_m) &= 128m - 78m = 50m, \\
M_1(P_2 \boxtimes P_m) &= 50m - 64.
\end{aligned}
$$

Consider Q_n on $n \geq 1$, then

$$
\begin{aligned}
M_1(Q_n) &= \sum_{u \in V(Q_n)} deg(u)^2 = n^2 \sum_{u \in V(Q_n)} 1 = n^2 2^n, \\
M_2(Q_n) &= \sum_{uv \in E(Q_n)} deg(u)deg(v) = n^2 \sum_{uv \in E(Q_n)} 1 = n^3 2^{n-1}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
M_1(Q_n \boxtimes P_m) &= (|V(P_m)| + 4|E(P_m)|)M_1(Q_n) + (|V(Q_n)| + 4|E(Q_n)|)M_1(P_m) \\
&\quad + M_1(Q_n)M_1(P_m) + 8|E(Q_n)||E(P_m)| \\
&= 2^n(9n^2m - 10n^2 + 4m - 6 + 12nm - 16n), \\
M_1(Q_n \boxtimes C_m) &= (|V(C_m)| + 4|E(C_m)|)M_1(Q_n) + (|V(Q_n)| + 4|E(Q_n)|)M_1(C_m) \\
&\quad + M_1(Q_n)M_1(C_m) + 8|E(Q_n)||E(C_m)| \\
&= m2^n(9n^2 + 12n + 4).
\end{aligned}
$$

Theorem 2.8. [20] *For the graphs G and H, we have*

$$
\begin{aligned}
M_2(G \boxtimes H) &= 3|E(H)|M_1(G) + 3|E(G)|M_1(H) \\
&+ 3M_1(G)M_1(H) + 2M_2(G)M_2(H) \\
&+ (6|E(H)| + 3M_1(H) + |V(H)|)M_2(G) \\
&+ (6|E(G)| + 3M_1(G) + |V(G)|)M_2(H).
\end{aligned}
$$

By the previous theorem,

$$
\begin{aligned}
M_2(P_n \boxtimes C_m) &= 3|E(C_m)|M_1(P_n) + 3|E(P_n)|M_1(C_m) \\
&\quad + 3M_1(P_n)M_1(C_m) + 2M_2(P_n)M_2(C_m) \\
&\quad + (6|E(C_m)| + 3M_1(C_m) + |V(C_m)|)M_2(P_n) \\
&\quad + (6|E(P_n)| + 3M_1(P_n) + |V(P_n)|)M_2(C_m) \\
&= 256mn - 414m, (m > 2).
\end{aligned}
$$

Consider Q_n on $n \geq 1$, then

$$\begin{aligned}
M_2(Q_n \boxtimes P_m) &= 3|E(P_m)|M_1(Q_n) + 3|E(Q_n)|M_1(P_m) \\
&+ 3M_1(Q_n)M_1(P_m) + 2M_2(Q_n)M_2(P_m) \\
&+ (6|E(P_m)| + 3M_1(P_m) + |V(P_m)|)M_2(Q_n) \\
&+ (6|E(Q_n)| + 3M_1(Q_n) + |V(Q_n)|)M_2(P_m) \\
&= 2^n(27mn^2 - 45n^2 + 18mn - 33n + \frac{27}{2}n^3m - 20n^3 + 4m - 8), \quad (7)
\end{aligned}$$

$$\begin{aligned}
M_2(Q_n \boxtimes C_m) &= 3|E(C_m)|M_1(Q_n) + 3|E(Q_n)|M_1(C_m) \\
&+ 3M_1(Q_n)M_1(C_m) + 2M_2(Q_n)M_2(C_m) \\
&+ (6|E(C_m)| + 3M_1(C_m) + |V(C_m)|)M_2(Q_n) \\
&+ (6|E(Q_n)| + 3M_1(Q_n) + |V(Q_n)|)M_2(C_m) \\
&= m2^n(\frac{27}{2}n^3 + 27n^2 + 18n + 4). \quad (8)
\end{aligned}$$

By replacing Q_n with Q_1 in the relations (7) and (8), we have M_2 of open and closed fences, as follow:

$$\begin{aligned}
M_2(Q_1 \boxtimes P_m) = M_2(K_2 \boxtimes P_m) &= 2(21m - 36 + 24m - 42 \\
&+ \frac{27}{2}m - 20 + 4m - 8) = 125m - 212, \\
M_2(Q_1 \boxtimes C_m) = M_2(K_2 \boxtimes C_m) &= 2m(\frac{27}{2} + 27 + 18 + 4) = 125m.
\end{aligned}$$

A connected graph is called a **self-centered** graph if all of its vertices have the same eccentricity. Then a connected graph G is self-centered if and only if $r(G) = D(G)$.

Theorem 2.9. [20] *Let G and H be self-centered graphs that $D(H) \leqslant D(G)$. Then*

$$\xi^c(G \boxtimes H) = 2r(G)(|E(G)||V(H)| + |E(H)||V(G)| + 2|E(G)||E(H)|).$$

One can see that $r(C_n) = [\frac{n}{2}]$. So, if $n \geq m$, then

$$\begin{aligned}
\xi^c(C_n \boxtimes C_m) &= 2r(C_n)(|E(C_n)||V(C_m)| + |E(C_m)||V(C_n)| + 2|E(C_n)||E(C_m)|) \\
&= 8nm[\frac{n}{2}].
\end{aligned}$$

Clearly, $r(Q_n) = n$, $|E(Q_n)| = n2^{n-1}$. Therefore,

$$\begin{aligned}\xi^c(Q_n \boxtimes C_m) &= 2r(Q_n)(|E(Q_n)||V(C_m)| + |E(C_m)||V(Q_n)| + 2|E(Q_n)||E(C_m)|)\\ &= n2^n(3mn + 2m) \quad if \quad n \geq \lfloor \frac{m}{2} \rfloor, \qquad (9)\\ \xi^c(Q_n \boxtimes C_m) &= 2r(C_m)(|E(Q_n)||V(C_m)| + |E(C_m)||V(Q_n)| + 2|E(Q_n)||E(C_m)|)\\ &= \lfloor \frac{m}{2} \rfloor 2^n(3mn + 2m) \quad if \quad n \leq \lfloor \frac{m}{2} \rfloor. \qquad (10)\end{aligned}$$

By replacing n with 1 in the relations (9) and (10), we have ξ^c of closed fence, as follow:

$$\xi^c(Q_1 \boxtimes C_m) = \xi^c(K_2 \boxtimes C_m) = \begin{cases} 10m & if \ \left[\frac{m}{2}\right] \leq 1, \\ 10m\left[\frac{m}{2}\right] & if \ \left[\frac{m}{2}\right] \geq 1. \end{cases}$$

3. Open Problem

We found the exact value of $\xi^c(G \boxtimes H)$, where G and H are self-centered graphs. A natural question arises here is if G and H are arbitrary graphs, then what is the value of $\xi^c(G \boxtimes H)$. If someone can find the answer, can calculate values of $\xi^c(Q_n \boxtimes P_m)$, $\xi^c(P_n \boxtimes P_m)$ and $\xi^c(P_n \boxtimes C_m)$ as a result.

Acknowledgement. The author would like to thank reviewer for careful reading and suggestions, which improved the manuscript.

Conflicts of Interest. The authors declare that there is no conflicts of interest regarding the publication of this article.

References

[1] A. R. Ashrafi, T. Došlić, A. Hamzeh, Extremal graphs with respect to the Zagreb coindices, *MATCH Commun. Math. Comput. Chem.* **65**(1) (2011) 85–92.

[2] A. R. Ashrafi, T. Došlić, M. Saheli, The eccentric connectivity index of $TUC_4C_8(R)$ nanotubes, *MATCH Commun. Math. Comput. Chem.* **65**(1) (2011) 221–230.

[3] A. R. Ashrafi, M. Saheli, M. Ghorbani, The eccentric connectivity index of nanotubes and nanotori, *J. Comput. Appl. Math.* **235** (2011) 4561–4566.

[4] G. G. Cash, Polynomial expressions for the hyper-Wiener index of extended hydrocarbon networks, *Comput. Chem.* **25** (2001) 577–582.

[5] G. G. Cash, Relationship between the Hosoya polynomial and the hyper-Wiener index, *Appl. Math. Lett.* **15** (2002) 893–895.

[6] A. A. Dobrymin, R. Entringer, I. Gutman, Wiener index of trees: theory and applications, *Acta Appl. Math.* **66** (2001) 211–249.

[7] A. A. Dobrymin, I. Gutman, S. Klavšar, P. Žigert, Wiener index of hexagonal systems, *Acta Appl. Math.* **72** (2002) 247–294.

[8] S. Gupta, M. Singh, A. K. Madan, Eccentric distance sum: a novel graph invariant for predicting biological and physical properties, *J. Math. Anal. Appl.* **275** (2002) 386–401.

[9] I. Gutman, Relation between hyper-Wiener and Wiener index, *Chem. Phys. Lett.* **364** (2002) 352–356.

[10] I. Gutman, N. Trinajstić, Graph theory and molecular orbitals. Total φ-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.* **17** (1972) 535–538.

[11] R. Hammack, W. Imrich, S. Klavšar, *Handbook of Product Graphs*, Second edition, CRC Press, Boca Raton, FL, (2011).

[12] H. Hosoya, Topological index. A newly proposed quantity characterizing the topological nature of structural isomers of saturated hydrocarbons, *Bull. Chem. Soc. Jpn.* **44** (1971) 2332–2339.

[13] A. Ilić, I. Gutman, Eccentric connectivity index of chemical trees, *MATCH Commun. Math. Comput. Chem.* **65** (2011) 731–744.

[14] M. H. Khalifeh, H. Yousefi-Azari, A. R. Ashrafi, The hyper-Wiener index of graph operations, *Comput. Math. Appl.* **56** (2008) 1402–1407.

[15] S. Klavšar, P. Žigert, I. Gutman, An algorithm for the calculation of the hyper-Wiener index of benzenoid hydrocarbons, *Comput. Chem.* **24** (2000) 229–233.

[16] D. J. Klein, I. Lukovits, I. Gutman, On the definition of the hyper-Wiener index for cycle-containing structures, *J. Chem. Inf. Comput. Sci.* **35** (1995) 50–52.

[17] M. J. Morgan, S. Mukwembi, H. C. Swart, On the eccentric connectivity index of a graph, *Discrete Math.* **311** (2011) 1229–1234.

[18] B. E. Sagan, Y. -N. Yeh, P. Zhang, The Wiener polynomial of a graph, *Int. J. Quant. Chem.* **60**(5) (1996) 959–969.

[19] V. Sharma, R. Goswami, A. K. Madan, Eccentric connectivity index: a novel highly discriminating topological descriptor for structure property and structure activity studies, *J. Chem. Inf. Comput. Sci.* **37** (1997) 273–282.

[20] M. Tavakoli, F. Rahbarnia, A. R. Ashrafi, Note on strong product of graphs, *Kragujevac J. Math.* **37**(1) (2013) 187–193.

[21] H. Wiener, Structural determination of paraffin boiling points, *J. Am. Chem. Soc.* **69** (1947) 17–20.

[22] G. Yu, L. Feng, A. Ilić, On the eccentric distance sum of trees and unicyclic graphs, *J. Math. Anal. Appl.* **375** (2011) 99–107.

[23] B. Zhou, Zagreb indices, *MATCH Commun. Math. Comput. Chem.* **52** (2004) 113–118.

[24] B. Zhou, Z. Du, On eccentric connectivity index, *MATCH Commun. Math. Comput. Chem.* **63** (2010) 181–198.

[25] B. Zhou, I. Gutman, Relations between Wiener, hyper-Wiener and Zagreb indices, *Chem. Phys. Lett.* **394** (2004) 93–95.

Mostafa Tavakoli
Department of Applied Mathematics
Ferdowsi University of Mashhad
P. O. Box 1159, Mashhad 91775, I. R. Iran
E-mail: m_tavakoli@um.ac.ir

Freydoon Rahbarnia
Department of Applied Mathematics
Ferdowsi University of Mashhad
P. O. Box 1159, Mashhad 91775, I. R. Iran
E-mail: rahbarnia@um.ac.ir

Irandokht Rezaee Abdolhosseinzadeh
Department of Applied Mathematics
Ferdowsi University of Mashhad
P. O. Box 1159, Mashhad 91775, I. R. Iran
E-mail: ir_rezaee899@stu.um.ac.ir