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The Role of Geometry of Yard in the Formation of the Historical Houses of Kashan

Ahmad Danaeinia⋆*, Mostafa Azad and Asma Khamehchian*

Abstract

Geometry is a basic element for establishing unity in Iranian architecture and is the important subject for architects for its discipline and rule. In this regard, the architecture of the house, in terms of its specific functional role, search to implement the principles and geometric rules in the best way and with understanding of the ratio and proportions achieve a correct of the harmony. Based on field study in 20 historical houses in Kashan, the study search to identify how the geometry is used in the architectural design of the historical houses of Kashan. This study is conducted through field study of 20 historical houses in Kashan to discover how geometry is used in the architectural design of Kashan historical houses. In this regard, the proportions of the area and the yard in these cells have been compared. The result shows that the yard follows the golden ratio as the house design basis. The 1.414 and 1.618 ratios have the highest frequency in the study samples. In addition, in terms of the level of occupation, the courtyard has occupied 20 to 40 percent of the building.

Keywords: Geometry, ratio, proportion, geometry of the area, geometry of the yard, traditional homes of Kashan.

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1. Introduction

Geometry and fitness are mathematical concepts that in the visual arts imply a good relationship between each other and the whole effect. The use of geometry

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and proportion is of special importance due to the creation of visual beauty in architecture and visual arts [5]. "Plato" wrote in the entrance his academy that anyone who knows geometry does not enter [23]. Traditional Iranian architects have special attention to the use of geometry and proportions in their works, and fit the space proportions of certain space proportions. In the works of Iranian artists, the ratio is widely used [23]. The traditional home of Kashan is exemplary example of the use of geometry and symmetry among the works of traditional architecture, and the need to preserve them and identify deep concepts lies in their hearts, study and study the different aspects of their physical and their meaning. The home in Iranian culture has a special place and in the traditional homes of Kashan, the courtyard plays a key role, fortunately a significant number of traditional Kashan homes remain from the old period, which provides a proper context for the study. The present study seeks to find the answer that what is the geometry and proportion of yard in the traditional homes of Kashan? And what is the connection between the proportions of the courtyard with the proportions of the house?

2. Methodology

In the present study, in order to compare the relation, proportions and geometry with the dimensions of traditional Kashan courtyard, with regard to the abundance of traditional houses, some houses have been selected as a case study. The base of sample selection is the oldness of the house and being comprehensive in the area. Data analysis is based on comparative and inductive method. The required information of the houses including the length and width of the building, the length and width of the courtyard, the total area of the house and the area of open space have been gathered through the studies and are compared and analyzed with specific proportions that include the golden proportion, the ratio of the 1.618, the 1.118 proportion, the radical proportion of 2 equal to the 1.414 and the radical proportion of 3 equal to the 1.73 which are known as Persian golden proportions.

3. Geometry in Architecture

The position of the geometry in the old architecture was to some extent that Buzjani (Abū al-Wafā', Muḥammad ibn Muḥammad ibn Yaḥyā ibn Ismā'īl ibn al-'Abbās al-Būzjānī, 10 June 940 – 15 July 998, was a Persian mathematician and astronomer. He made important innovations in spherical trigonometry, and his work on arithmetics for businessmen contains the first instance of using negative numbers in a medieval Islamic text) held meetings and practical workshops in Baghdad, half of which were architect and half of mathematicians. In these meetings, he tries to create a connection between art and mathematics by challenging artists and mathematicians with the design of common problems [7]. O'kane's

study reveals that in the design and implementation of buildings, the knowledge of geometry and scale systems and networks are used. On the other hand, this mixture is so high that there is a variety of books that have been written in this field, including Ghiyāth Al-Dīn Kāshī (Ghiyāth Al-Dīn Jamshīd Mas'ūd al-Kāshī, 1380, Kashan, Iran-1429, Samargand, Transoxania, was a Persian astronomer and mathematician) which is about the size and volume of architecture. In all development steps, the relationship between geometry and structure is observed [1]. The recent studies on the relationship between mathematics and geometry and Iranian art suggest that Iranian Muslim artists in the middle ages have great advances in mathematics and geometry [19]. Geometry is a measure of size and is defined as knowledge that is a mathematical relation between points. The weights determine levels and volumes and show the proportions between them and their derivatives. The science of geometry, like all other sciences, derives from observation and experience and is closely related to human economic needs; the word geometry in European languages has Greek roots, and means survey (land measurements). Geometry and geometric concepts are the off spring of human experience and necessity, and on the other hand, they are tested and used in practical science. The geometry is always developed in both practical and theoretical aspects, and it can be said that the axis is the path of geometry, but in terms of differences in results and differences in the way each axis is used is independent of one another. What is known as geometry is two parts, theoretical geometry and practical geometry, theoretical geometry of lines, planes and formations, so the same geometry is on paper, practical geometry on geometry and metal and brick and other formations [14]. The knowledge of geometry is practical in the subject of architecture; practical or empirical geometry is set of techniques and rules that help designers to create or organize the design. Principles of mathematics are among the lands and sciences. It is obvious that the properties of the square, circular, symmetrical geometric shapes, angles, parallel lines, etc. [1]. Of course, square specifications, circles, geometric shapes, angles, parallel lines, etc. are not inconsistent in geometry, but the method of understanding and using of geometry is different. The ways of drawing, method of finding fault, ways of proving authenticity, ways to design and select proportions which are used in architecture and related arts and crafts and the type of supply and extension of the principal components indicate that the Iranian geometry is distinguished from European geometry [1]. Geometry is the ability of the department in design and is the architectural tool for how to communicate and link between forms and masses. The discipline between elements of the building, integrity, creation of creativity and communication with the audience can be achieved through this knowledge. So, geometry is for the architect as well as art. It is also the main pattern for design and creation of work and is a key element to build the connection between the architect's mind and what is to be created [15]. Therefore, in addition to its aesthetic merit, Islamic geometric design is renowned for its mathematical sophistication, constituting the most highly developed chapter in cultural symmetry studies [22]. The subject of geometry from ancient times has been studied by researchers and has a fundamental root among

the sciences, whose symbols abound in ancient Persian and non-Persian texts. For example in research of Al-Farabi (Abū Naṣr Muḥammad ibn Muḥammad al Fārābī, 14 December 872 – 12 January 951, in Islamic philosophical tradition, he is known with the honorific "The Second Teacher", after Aristotle being known in the east as "The First Teacher")[14] and Efendi [12] and recently, Halabi [15], Daneshpazhoh [11], Ansari et al. [3] in the investigating the historical process of arithmetic arrangement systems with emphasis on practical considerations and aesthetics, proportions and styles, and their importance. Amirkhani et al. [2] investigating the metamorphism of the Timche symmetry in the Qajar period; the proportions in the Timcheh were investigated. Furthermore, Attarian et al. [4] investigating the parameters of the yard of the mosques in the Safavid period of Isfahan, and Porahmadi et al. [21] studied the proportions of the length to width ratio courtyards and rooms in traditional Yazd houses a test for Pirniya about Iranian golden tent. In geometry and ornament in the Islamic architecture, Necipoglu, described an example of the traditional application of geometry and proportions [17]. Mehdizade Seraj et al. [16] also pointed to the use of proportions in decorations in applying normal triangles in mathematical calculus and implement geometry in the construction and implementation traditional Iranian architecture. However, the monuments built and lasted in Iran; they are honest proof about the effect of math in architecture [10]. Figure 1 shows geometrical pattern in Ghiasiyeh school in Khaf, Khorasan, Iran and Figure 2 shows ratio, proportions and modular system in Furomad mosque in Sabzevar, Iran. Elegant geometry used in the facade of these monuments, show that the exact relationship between architecture and mathematics.

Figure 1: General view (left) and detail view (right) of geometrical pattern in Ghiasiyeh school, Khaf, Khorasan, Iran.

Figure 2: The ratio, proportions and modular system in Iranian architecture, Forumad mosque, Sabzevar, Iran.

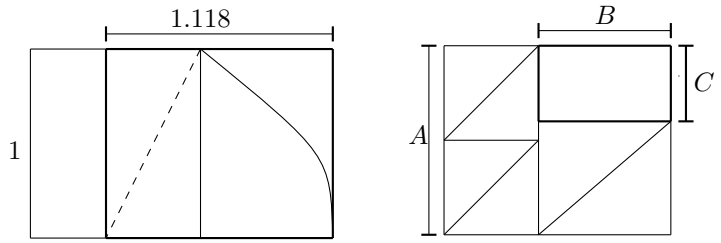
4. Ratio and Proportion

In the works of Iranian artists, the ratios are expertly woven. When we look at these works, we feel that we are in a space in the mystic ether. The ratios of the carpets and the volume of precious books that have been properly secured and preserved from the old days till now [23]. The fitness is the relative and relative relation between the different components and all elements. The measure of the size of two things, a ratio, and fitness, or fitness, is defined as the equality of these ratios [1]. The geometry deals with the basic proportions, as far as possible. The dimensions of the function are interdependent and multiple multiples of each other, following the need of the proportions [3]. Each coordinate system has the specific proportions that are established between the components together with each other. In the architectural area, the proportions of the comparison ratios range from different quantities of differentiation. Whereas, proportions depend on the science of geometry and mathematics in its place and in its specialized form, it has an undeniable value at the level of understanding art and is considered fundamental considerations. Moreover, the discussion of proportions in general and in particular in the literature of contemporary art and architecture has a strange effect [4]. In architecture, the proportions of the comparison ratios range from different quantities of differentiation. Whereas, proportions depend on the science of geometry and mathematics in its place and in its specialized form, it has an undeniable value at the level of understanding art and is considered fundamental considerations [4].

Concerning the proportions of art, the fundamental purpose of all theories is to feel the discipline between the parts of an intuitive combination. Although the proportions in the first look may not appear to the viewer, the visual composition has emerged, in a sequence of continuous experiments, the creator of a sense of beauty [18]. Examples of popular proportions that have been used both in the Iranian architecture and in the European architecture are the golden proportion with the 1.618 number, the symmetry of the 1.414 (castle of Takhte Jamshid and Apadana, Shiraz, Iran), the proportionality of the 1.73 (castle of Kasra, Tisfoon), the proportionality of 1.118 (castles of Sarvestan, Fars, Iran and Taq-i Kisra, Tisfoon, Iraq) [18]. The traditional architects used these ratios to create the proportions and their system is based on the calculation of complex mathematical and radical numbers. The correct proportions that different periods of time were created in the ancient civilizations were created using calculated measures, in the designs of the creative architectural designs, with a sense of beauty [13]. The golden ratio that is seen in the shapes of some plants, flowers, viruses, DNA, shells, planets and galaxies is equal to the fraction $\frac{1+\sqrt{5}}{2}$ and almost equal to 1.618 [6], as well as the 1.118 ratio obtained from the gold proportion achieved in pre-Islamic architectural proportions, this ratio can be found on the longer and the width of the Sarvestan palace and the palace of the Taq-i Kisra, palace from the Sassanian palace [9]. The use of length which is proportional to certain length, or the scale system in which the length ratio is equal to the specified ratios. Applied geometrical proportions are as follow:

$$\sqrt{2} = 1.414, \qquad \sqrt{0.5^2 + 1^2} = 1.118, \qquad \frac{1+\sqrt{5}}{2} = 1.618, \qquad \sqrt{3} = 1.73.$$

In order to obtain these ratios, the graphic method can be performed at Figures 3 and 4.



$1 : 1.118$

$\frac{B}{A} = \frac{C}{B} = 0.618$

$0.618 + 0.5 = 1.118$

$\sqrt{0.5^2 + 1^2} = 1.118$

Figure 3: 1.118 ratio [8].

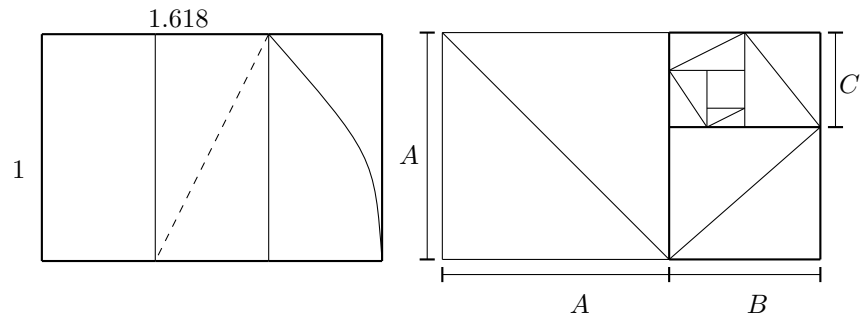


Figure 4: 1.618 ratio [8].

Traditional architects used different ways to implement these proportions, one of which is the most accurate of which is the norm of triangle. The norm of triangle tool, in addition to dividing the line, is readily employed by an architect to square, square root, and other mathematical functions, which have no other solution except by difficult mathematical methods. The norm triangles have the right angles that are special between their components and are used in many architectural and related works [16] the method of the norm triangle is based on practical and efficient geometry for architects, carpenters and other artists, which effectively express the application and adaptation of geometry to an artist's instrument without approximation and error [16].

5. Earth Geometry (The Field of Construction) at Case Studies

The architecture of Kashan as a prominent indication of Iranian-Islamic architecture suggests that the city of Persian-Islamic architecture and architecture has never been alien to nature. Architecture in this city was forced to intervene in nature to create the building, but not only to destroy it, but also to balance the existence of the balance. In addition, to create the city and architecture with the rule, all sciences such as geography, geometry and irrigation were used. The water order and the soil order have a great impact on the shape of the physical structure of the city [20]. In the formation of Kashan houses, attention to nature: the proper utilization of the earth's capacities is proportional to the order of water and soil order (nature)as the most important factors shaping the architecture of the house. In 20 case studies, the relationship between earth and fabrication has

been investigated and the proportions of these two have been analyzed. Table 1 shows the ratio of length to the area in the area of the investigated cells.

Table 1: Study of the ground geometry in case studies.

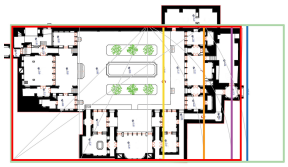
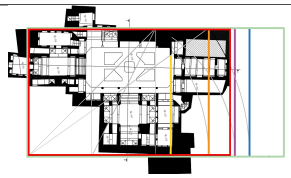
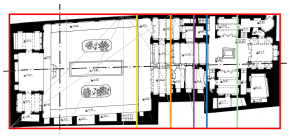
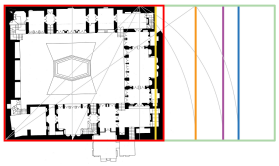
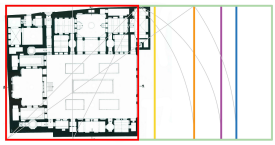
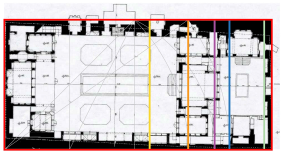
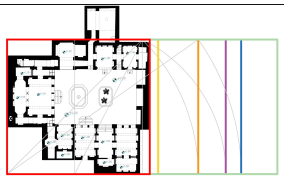
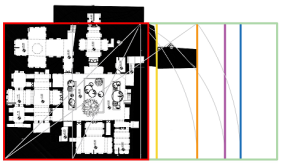
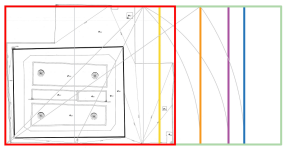
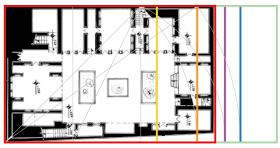
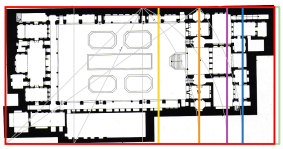
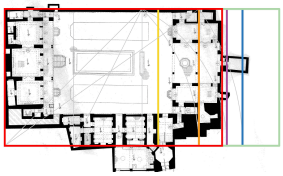
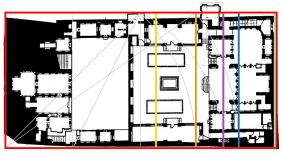
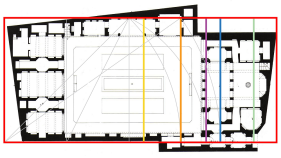
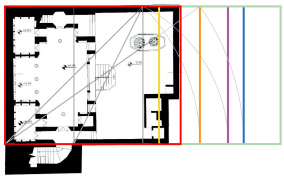
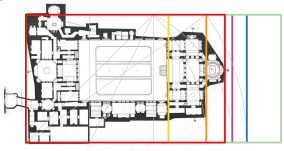
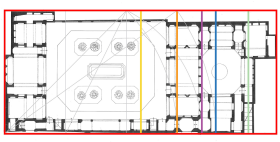
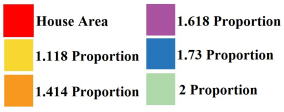
Number	Case study	Earth geometry system			
		Length	Width	Length to width	Approximation to specific proportions
1	Manouchehri	52	30	1.73	1.73
2	Lajevardi	50	30	1.66	1.618
3	Kaj	26	18	1.46	1.414
4	Sodori	30	25	1.2	1.118
5	Atarha	34	34	1	1
6	Hashemian	56	27	2	2
7	Soaei	25	18	1.39	1.414
8	Ostovar	32	25	1.28	1.414
9	Balalzadeh	46	38	1.21	1.118
10	Hoseini	70	38	1.84	1.73
11	Vosgha	20	13	1.5	1.618
12	Saleh	66	34	1.94	2
13	Zarkar	46	30	1.53	1.618
14	Mazandarani	54	26	2	2
15	Bakochi	40	19	2	2
16	Tahami	40	18	2.22	-
17	Sadeghi	37	13	2.84	-
18	Shabani	20	16	1.25	-
19	Sharifian	59	37	1.57	1.618
20	Roeintan	38	17	2.17	-

The case studies (Table 2) show that the number of households is different. In other words, the size of the building is a function of the size of the household and is close to the ratio of the 1.618 (golden ratio), 1.414, 1.118, 1.73 and 2. However, despite the limitation of the earth dimensions and its non-geometrical shape in the design for the architect, the geometry of the building is subject to rules and norm and has modulus. Traditional architects, with the index of placing the courtyard and regulating it, have changed the spaces to a harmonious geometry and used spaces that were created in the corners, were used for ancillary and service spaces.

6. Yard Geometry in Case Studies

One of the properties of the courtyard in Iranian house, geometrical arrangement and the use of simple shapes will be familiar and legible to provide a calm environment for residents lives. In addition to establishing unity among elements,

Table 2: Presents of the proportions of the examined cells on their plan.

 1. Manouchehri	 2. Lajevardi	 3. Taj
 4. Sodori	 5. Atarha	 6. Hashemian
 7. Shoaei	 8. Ostovar	 9. Balalzadeh
 10. Hoseini	 11. Vosgha	 12. Saleh
 13. Zarkar	 14. Mazandarani	 15. Bakochi
 16. Tahami	 17. Sadeghi	 18. Shabani
 19. Sharifian	 20. Roeintan	

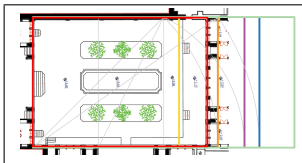
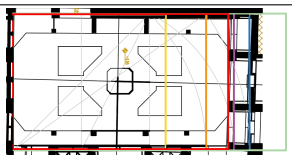
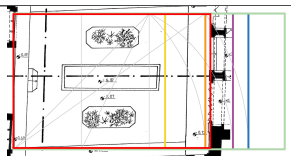
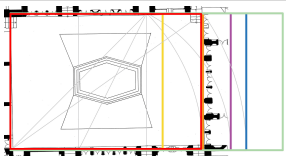
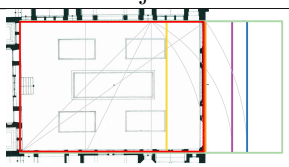
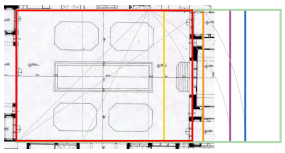
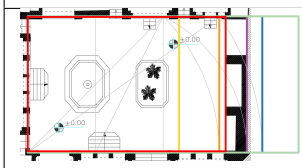
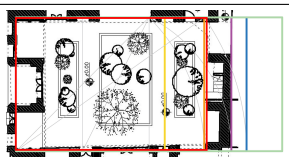
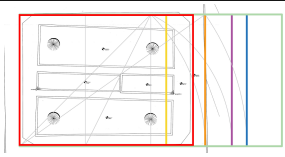
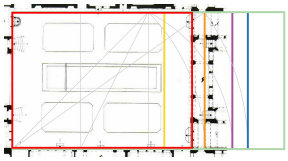
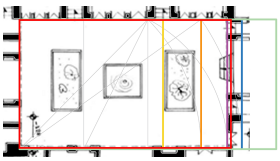
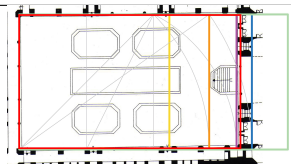
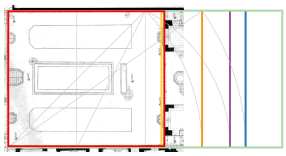
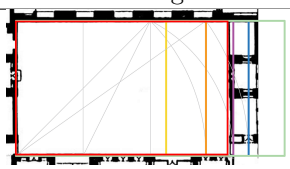
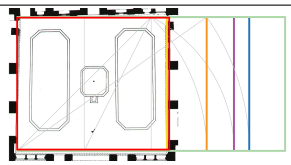
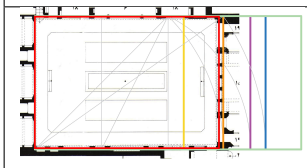
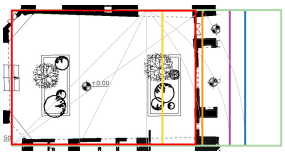
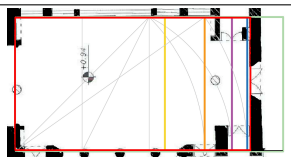
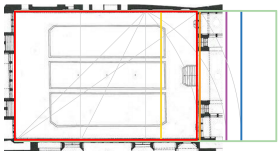
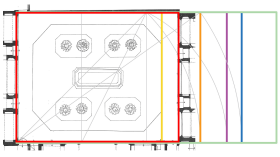
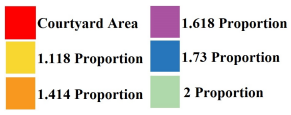
the courtyard is very effective in building geometry for the entire building. In this regard, the geometry of the yard as central part and the spinal column of Kashan houses are shaping the geometrical system of the whole house. Table 3 investigates the geometry of the yard in samples. The existence of the proportions and proportions of the proportions in the courtyard has been investigated by measuring the length of width and comparison with the gold ratios (Figures 1 and 2). The proportions are examined in the courtyard of the houses. The results of the investigations are presented in Table 3.

Table 3: Geometry of the yard in case studies.

Number	Case study	Earth geometry system			
		Length	Width	Length to width	Approximation to specific proportions
1	Manouchehri	24.8	18	1.38	1.414
2	Lajevardi	21.2	13.5	1.57	1.618
3	Kaj	11.5	8	1.43	1.414
4	Sodori	20	14	1.4	1.414
5	Atarha	23.5	16.7	1.4	1.414
6	Hashemian	26	20	1.3	1.414
7	Soaei	14.3	9.5	1.5	1.414
8	Ostovar	13	9	1.44	1.414
9	Balalzadeh	27	21	1.29	1.414
10	Hoseini	28	22	1.27	1.414
11	Vosgha	11	6.7	1.645	1.618
12	Saleh	32	19.4	1.64	1.618
13	Zarkar	26	22	1.18	1.118
14	Mazandarani	26	16.5	1.57	1.618
15	Bakochi	16	13.5	1.18	1.118
16	Tahami	20.5	14.5	1.41	1.414
17	Sadeghi	11.5	8.5	1.35	1.414
18	Shabani	14	8	1.75	1.73
19	Sharifian	29.3	20.4	1.43	1.414
20	Roeintan	17.95	13.85	1.29	1.414

The investigation of ratio and proportion in the yard shows that more than half of the samples have 1.414 ratio and four houses have a 1.618 ratio in yard, one case has 1.73, and two cases have 1.118 ratio in yard. This ratio is close to the ratio of 1.414 which is an important feature in the geometrical system of Iranian houses (Table 4).

Table 4: The proportions of the yard in plan of case studies.

 1. Manouchehri	 2. Lajevardi	 3. Taj
 4. Sodori	 5. Atarha	 6. Hashemian
 7. Shoaei	 8. Ostovar	 9. Balalzadeh
 10. Hoseini	 11. Vosgha	 12. Saleh
 13. Zarkar	 14. Mazandarani	 15. Bakochi
 16. Tahami	 17. Sadeghi	 18. Shabani
 19. Sharifian	 20. Roeintan	

7. Main Results

There is a direct relationship between the geometry of the yard and the geometry of the earth, and both follow certain rules in a logical order. The order of the court is based on the design and discipline of other spaces. In all samples, the percentage of open space shows that %20 to %40 of the land is open. Table 5 shows the comparison of the proportions of the earth and the yard.

Table 5: Comparison of the geometry of the earth and geometry of the yard in the case study.

Number	Case study	Total space	Open space surface	The ratio of open space to total space	The ratio of total space	The ratio of open space	Comparative of proportion in total and open space
1	Manouchehri	1440	480	33	1.73	1.414	-
2	Lajevardi	959	288	30	1.66	1.57	Near
3	Kaj	460	171	37	1.46	1.43	Near
4	Sodori	830	282	34	1.2	1.4	Near
5	Atarha	1180	400	33	1	1.4	-
6	Hashemian	1450	660	45	2	1.3	-
7	Soaei	515	138	27	1.39	1.2	Near
8	Ostovar	580	125	22	1.284	1.44	-
9	Balalzadeh	1464	570	39	1.21	1.29	Near
10	Hoseini	2465	640	26	1.84	1.27	-
11	Vosgha	256	74	29	1.5	1.64	Near
12	Saleh	1936	751	39	1.94	1.64	-
13	Zarkar	1498	575	38	1.53	1.18	-
14	Mazandarani	1365	440	32	2	1.57	-
15	Bakochi	800	216	27	2	1.38	-
16	Tahami	729	297	40	2.22	1.41	-
17	Sadeghi	540	94	18	2.84	1.35	-
18	Shabani	350	115	33	1.25	1.75	-
19	Sharifian	2214	603	27	1.57	1.92	-
20	Roeintan	645	250	38	2.17	1.29	-

8. Conclusion

Geometry is the most important element of architectural components in the historical homes of Kashan. Attention to proportions and respect of proportions in architecture creates space order. In Kashan historical houses, observing proportions is seen in different sections. The dimensions of the ground are selected according to the needs of the user and the social, cultural and economic situation of the people, but with centrality of the courtyard in design of the home components, complete integration between the ground and its various parts. The geometrical shape of the central space of all the houses is rectangular and its stretching is in

the direction of the overall building and in full following the climate. The courtyard [space in 13 of the case studies (%65) has 1.414 proportions, in 4 of the case studies (%20)], has a 1.618 proportion, in 1 of the case studies has a 1.73 ratio and in two of them is 1.118.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] L. Aboalghasemi, *The Norm of Formation in Islamic Architecture of Iran*, in the endeavor of Mohammad Y. Kiani, Samt Publication, 2005.

[2] A. Amirkhani, P. Baghaii and M. R. Bemanian, Investigating the metamorphism of the Timche symmetry in the Qajar period, *Fine Arts* **37** (2014) 39 – 48.

[3] M. Ansari, H. Okhovat and A. A. Taghvaee, Investigating the historical process of arithmetic arrangement systems with emphasis on practical considerations and aesthetics, *Ketab-e Mah-e Honar* **151** (2011) 46 – 57.

[4] K. Attarian, K. Momeni and Z. Masoudi, Investigating the parameters of the Yard of the mosques in the Safavid period of Isfahan, *Motaleat-e Tatbighi-e Honar* **10** (2014) 67 – 81.

[5] M. R. Bemanian, H. Okhovat and P. Baghaii, *Application of Geometry and Proportions in Architecture*, Tahan Publication, Tehran, 2010.

[6] M. R. Bemanian, Introduction to peymoon's role and application in Iranian architecture, *Modares-e Honar* **1** (2001) 1 – 10.

[7] A. Brooke, *Geometric Patterns in Islamic Art*, Translate: B. Zabihian, Maziar Publication, Tehran, 2007.

[8] R. L. Brown, *H. H. Richardson and the Golden Section Proportions in his Architecture*, Richard Brown Publication, New York, 2010.

[9] T. Chand, *The Impact of Islam on Indian Culture*, Translate: A. Pirnia and A. Osmani, Pajang Publication, Tehran, 1994.

[10] A. Danaienia and M. Fattahpoure, The relationship between architecture and symmetry in Persian architecture: A case study of Sheikh Lotfollāh mosque in Isfahan, *Symmetry: Culture and Science* **29** (2018) 423 – 439.

[11] M. T. Daneshpajoh and I. Afshar, *Mojmal Al Hikmah*, Institute of Humanities and Cultural Studies Publication, Tehran, 1996.

[12] J. Efendi, *Risale Memariye*, Translate: M. Ghaiomi, Matn Publication, Tehran, 2010.

[13] M. Falamaki, *The Formation of Architecture in the Experiences of Iran and the West*, Faza Publication, Tehran, 1991.

[14] A. M. al-Farabi, *Ihsā al-'ulūm*, Translated from the Arabic by Khadyūjim, Scientific and Cultural Publication, Tehran, 1969.

[15] M. Ghochani and M. Taji, *Geometric Structure Analysis Karbandi and Compare it with Geographic Fractal*, International Conference on Modern Studies in Civil Engineering, Architecture and Urban Planning, Mashhad, Sajjad Industrial University, 2016.

[16] F. Mehdizade Seraj, F. Tehrani and N. Valibeyg, Applying normal triangles in mathematical calculus and implement geometry in the construction and implementation traditional Iranian architecture, *Maremat & Memari-E Iran* **1** (2010) 15 – 26.

[17] G. Najiboghlo, *Geometry and Decoration in Islamic Architecture*, Translate: M. Ghaiomibidhendi, Rozane Publication, Tehran, 1999.

[18] A. Noghrekar, *Theoretical Foundations of Architecture*, Payamnoor University Publication, Tehran, 2009.

[19] J. Peter Lu and J. Paul Steinhardt, Decagonal and quasi-crystalline tilings in medieval Islamic architecture, *Science* **315** (2007) 1106 – 1110.

[20] A. Pourahmad and A. Vafaii, The effect of modernism on the physical-spatial structure of the Iranian-Islamic city (case study: Kashan), *Motaleat-e- Shahr-e- Irani-Eslami* **28** (2016) 63 – 76.

[21] M. Porahmadi, M. Yusefi and M. Sohrabi, The ratio of length to width in the main spaces of traditional houses of Yazd: A test for Pirnia's statement on Iranian golden rectangle, *Fine Arts* **47** (2010) 69 – 77.

[22] P. Saltzman, Quasi-periodicity in Islamic geometric design, In: K. Williams and M. Ostwald (eds) *Architecture and Mathematics from Antiquity to the Future*, Birkhäuser, Cham, Springer, Switzerland, 1 (2015) pp. 585 – 602.

[23] A. Sharaf Al Din, *Pleasant Geometry*, Madrese Publication, Tehran, 2003.

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